Few-nucleon reactions and the nuclear interaction

TNET Workshop

Guildford (UK)

M.Viviani

INFN - Pisa (Italy)

Istituto Nazionale di Fisica Nucleare

SEZIONE DI PISA

** Summary **

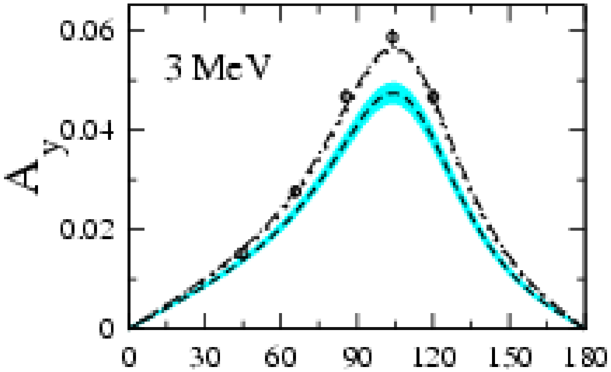
- NN interaction
- HH method for A=3,4 nuclear systems
- Bound states & Form Factors
- p- ^{3}He scattering
- Developments & conclusions

NN interaction (1)

- Realistic (phenomenological) potentials
    - Argonne V18 [Wiringa et al, 1995]
    - CD Bonn [Machleidt, 2001]
    - Nijmegen [Stoks et al, 1994]
  - ≈40 parameters fitted to NN data

- Underbinding of $A \geq 3$ nuclei

- A_y puzzle

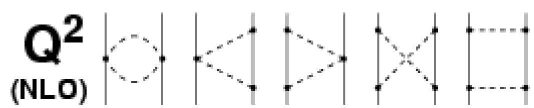


NN interaction (2)

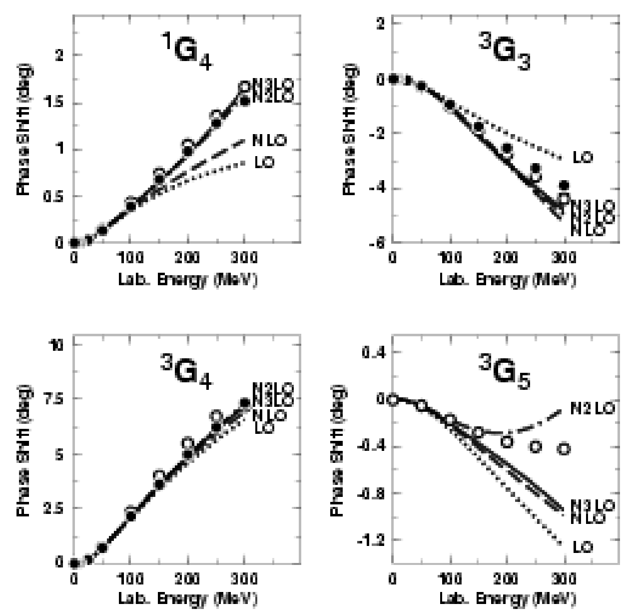
- Effective field theory based on chiral symmetry
  - [Weinberg 1991, van Kolck 1994]

$$\mathcal{L} = \overline{N}\left[i\partial_0 - \frac{1}{4 f_\pi^2}\vec{\tau}\cdot(\vec{\phi}_\pi \times \partial_0 \vec{\phi}_\pi) - \frac{g_A}{2 f_\pi}\vec{\tau}\cdot(\boldsymbol{\sigma}\cdot\nabla)\vec{\phi}_\pi + \ldots \right] N$$

- Presence of the chiral two-pion exchange component
  - [Rentmeesteer et al, 1999]



- N^3LO potential
  - [Epelbaoum et al, nucl-th/0405048]
  - [Emtem & Machleidt, 2003]



NN interaction (3)

- Low-momentum potentials
  - [Bogner, Kuo & Schwenk, 2003]

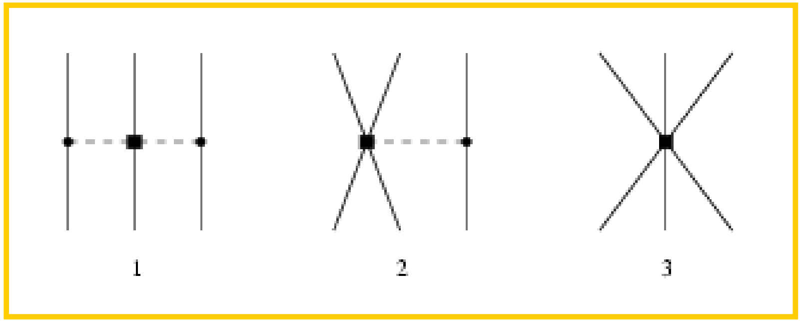
$$\text{low energy states} \, |n> \text{ with } k < \Lambda$$

$$P = \sum_{n}^{k<\Lambda} |n><n| \qquad PH_{low\,k}\,P\Psi = EP\Psi$$

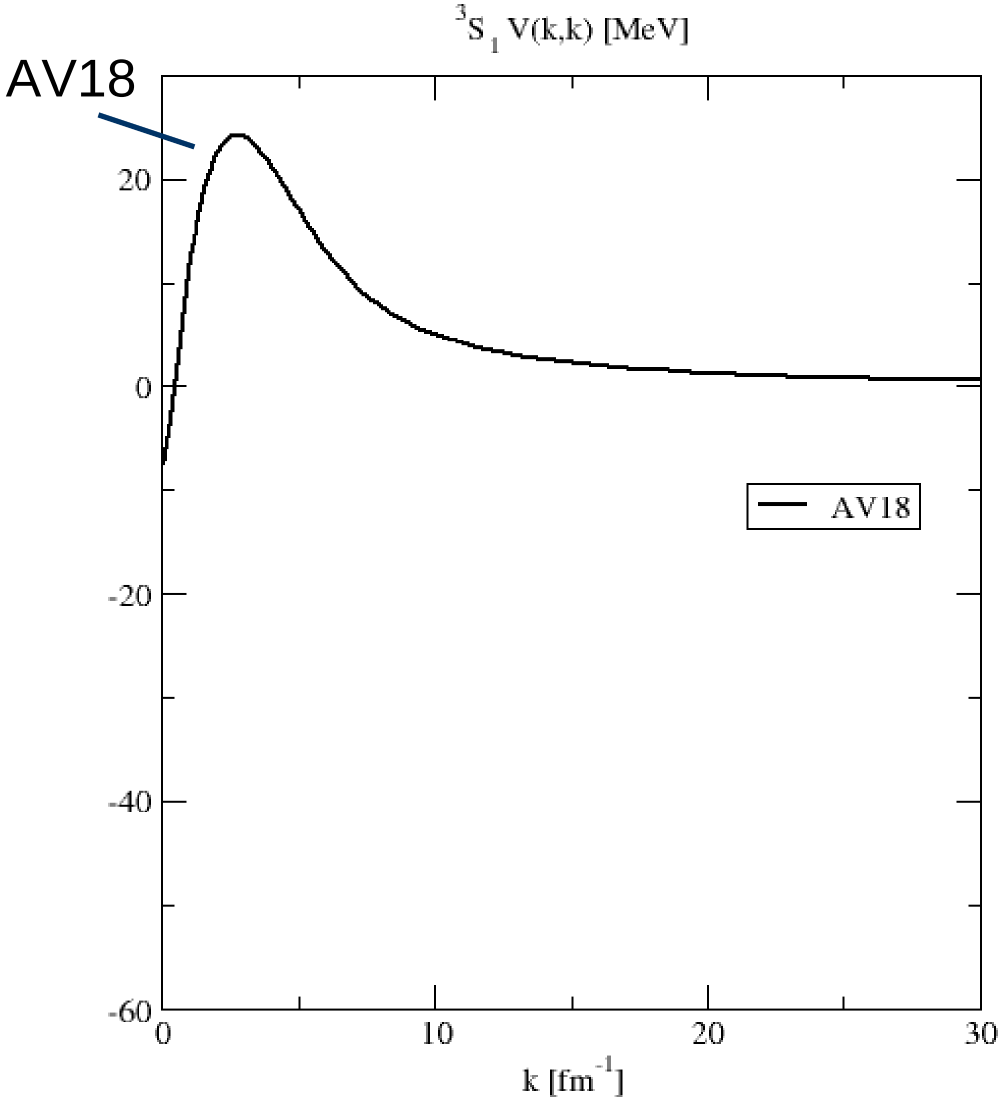
  - The new effective Hamiltonian is operative only within the low-energy model space

3N interaction

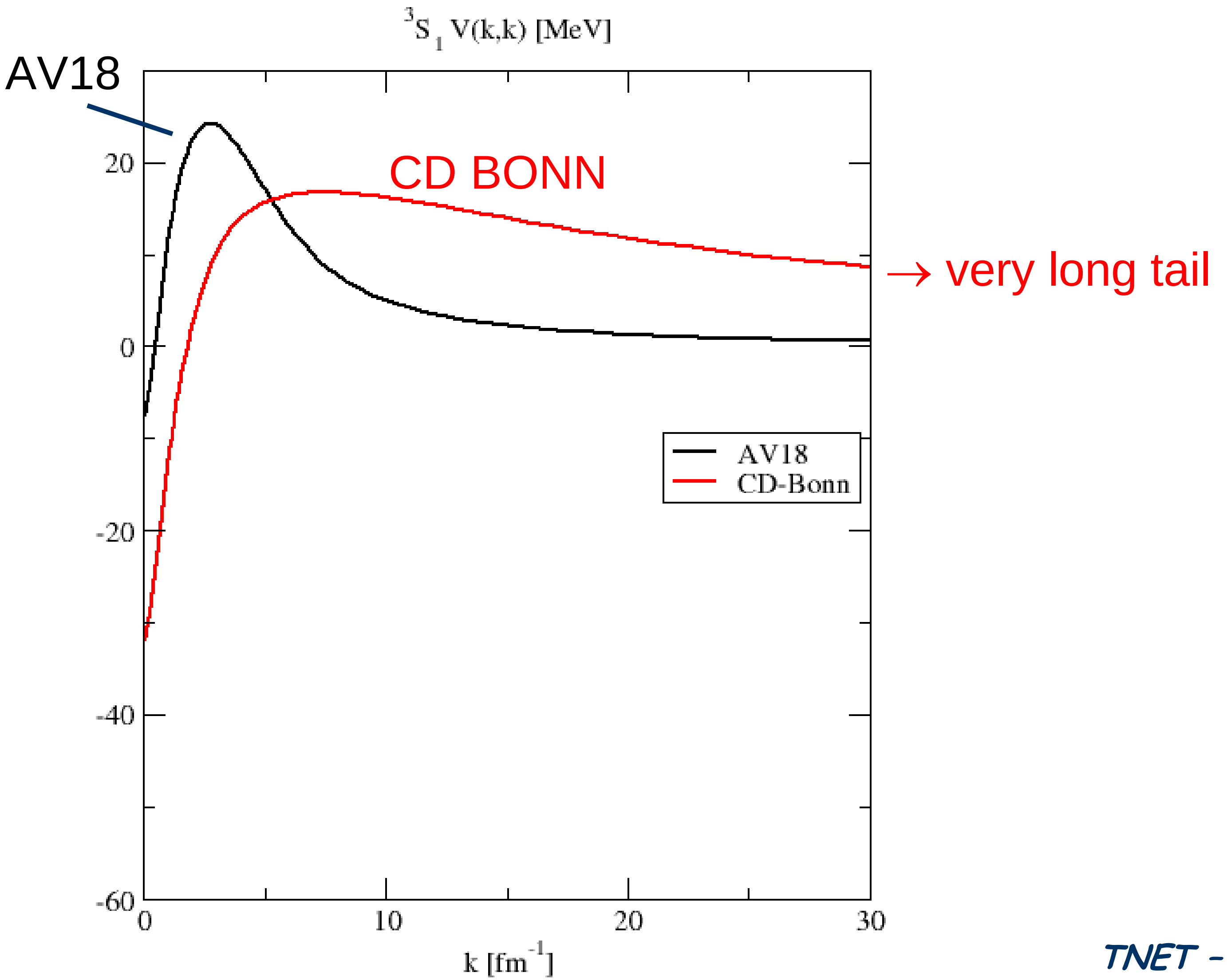
- "Old models"
  - Brazil & Tucson Melbourne [Friar et al, 1999]
  - Urbana [Epelbaoum et al, 2002]
- new proposed models
  - Illinois (3π exchanges) [Pieper et al, 2001]
  - Chiral symmetry
    - [Friar et al, 1999]
    - [Epelbaoum et al, 2002]



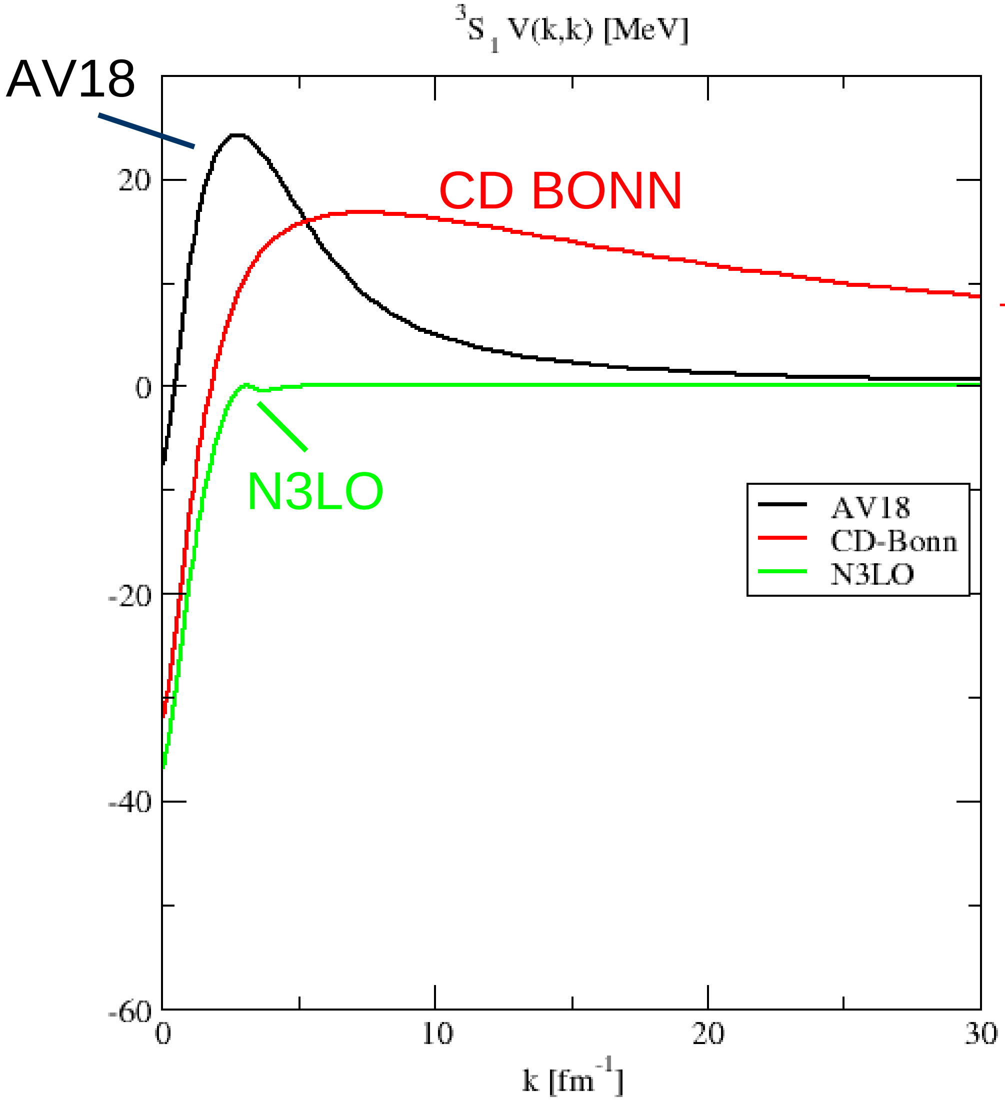
NN potentials in p-space



NN potentials in p-space



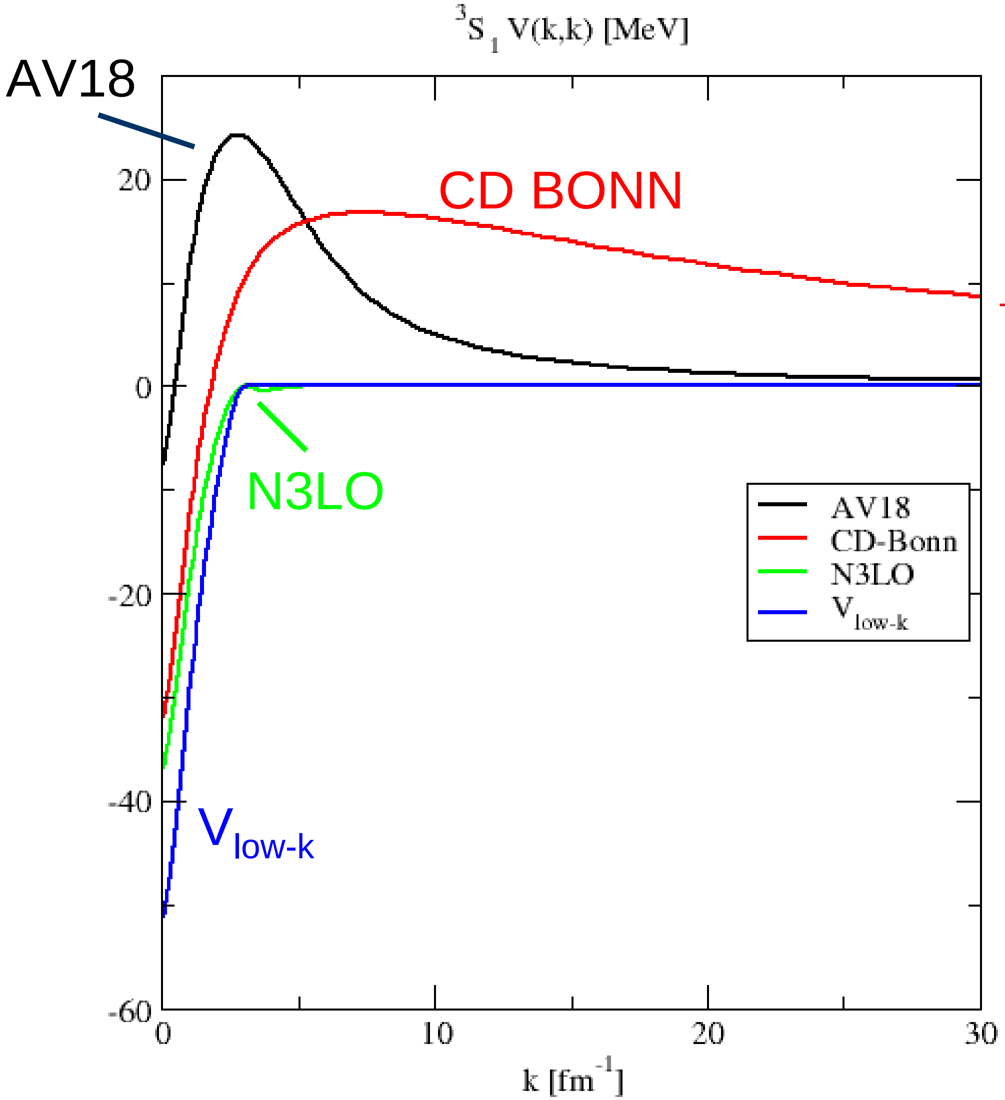
NN potentials in p-space



→ very long tail

$V_{N3LO}(k,k')\approx 0$ for $k,k'>5$ fm^{-1}

NN potentials in p-space



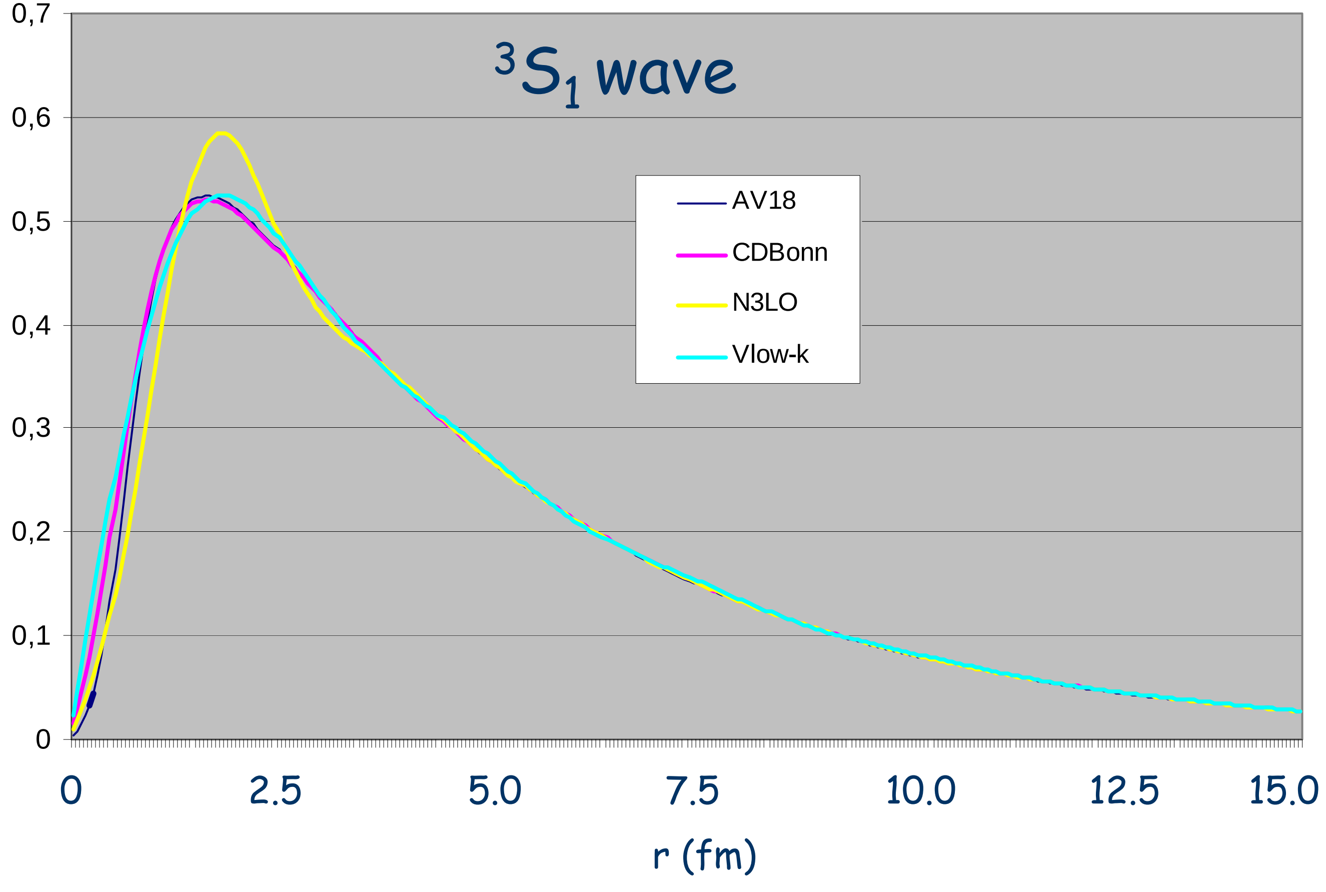
→ very long tail

$V_{N3LO}(k,k')\approx 0$ for k,k'>5 fm^{-1}

$V_{low-k}(k,k')=0$ for k,k'>2.1 fm^{-1}

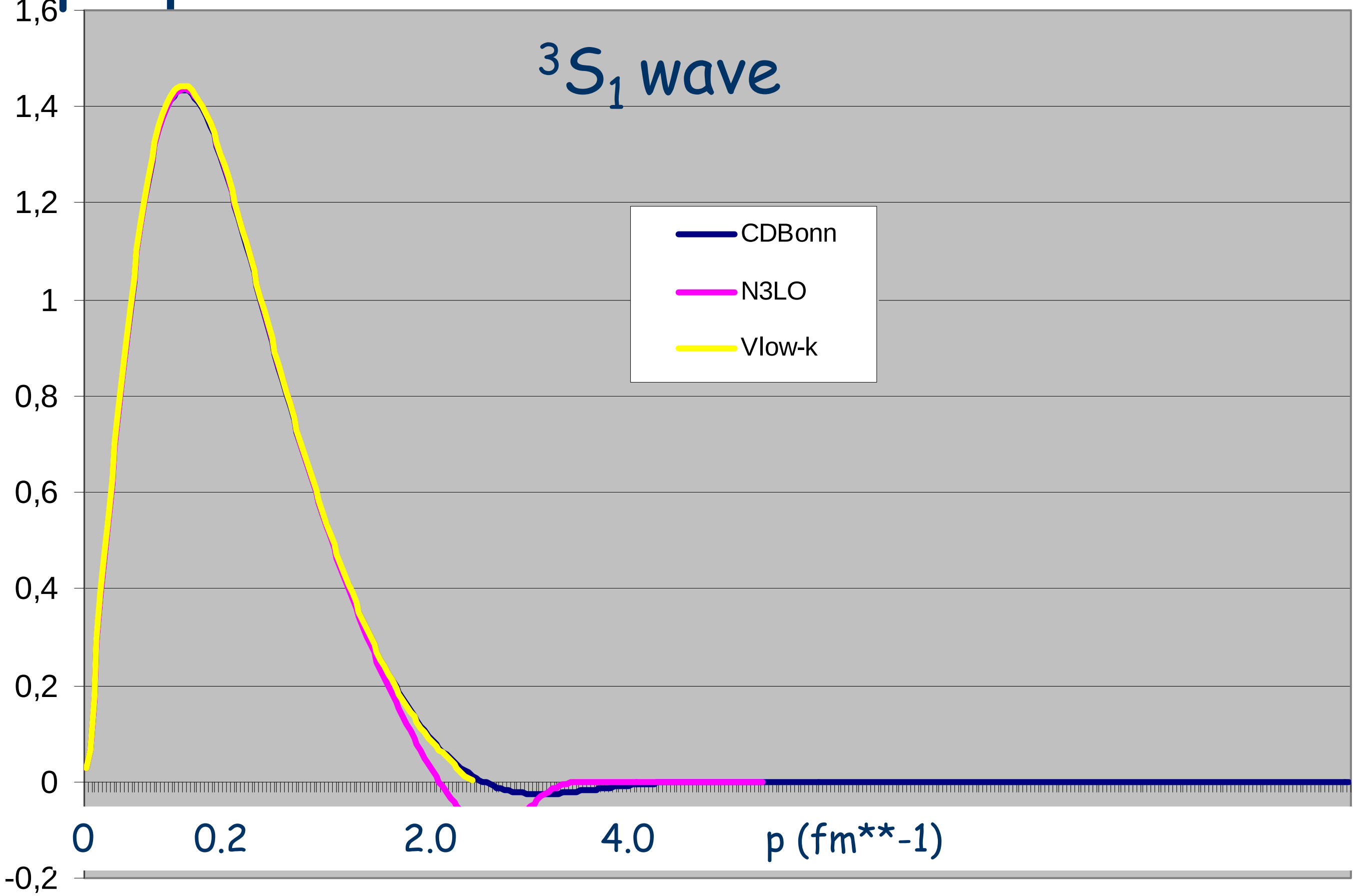
Deuton wave function

- In r-space:



Deuton wave function

- In p-space:



The A=3 & 4 systems as a laboratory

- Test of new NN forces
  - "tuned" using 2N and 3N systems
- Electro-magnetic & weak transitions
  - Transition operators $\Leftrightarrow$ interactions
  - Electron scattering & radiative capture
- Study of the 3N & 4N dynamics
  - Reactions of astrophysical interest
  - New effects (resonances, …)

$\Rightarrow$ We need accurate and reliable techniques

HH method *[Kievsky, Rosati, Marcucci, MV]*

- Expansion of the wave functions in the HH basis
  - Correlated version [NP **A551**, 241 (1993)]
- Variational methods (Rayleigh-Ritz or Kohn-Hulthen)
- HH functions= eigenfunctions of a part of the kinetic energy operator

$$\Lambda^2(\Omega) Y_{K,\nu}(\Omega) = -K(K+3A-5) Y_{K,\nu}(\Omega)$$

- For bound states: convergence properties with respect to *K* known
  - [Schneider, 1972]

•Zernike & Brinkman, 1935
•Simonov, 1967
•Efros, Zhukov, …, 1972
•Fabre de la Ripelle, 1983

- It can be applied equally well in r- and p-space
  - Bound states
  - Scattering states (in progress)

HH & CHH method (1)

- Jacobi vectors $\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3$
- Hyperangular variables

$$\rho, \Omega = (\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \phi_2, \phi_3)$$

$$\rho = \sqrt{(\vec{\xi}_1)^2 + (\vec{\xi}_2)^2 + (\vec{\xi}_3)^2}$$

$$\begin{cases} \cos\phi_3 = \xi_3 / \rho \\ \cos\phi_2 = \xi_2 / \sqrt{(\xi_1)^2 + (\xi_2)^2} \end{cases}$$

- Kinetic energy

$$T \to -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial\rho^2} + \frac{8}{\rho}\frac{\partial}{\partial\rho} - \frac{\Lambda^2(\Omega)}{\rho^2}\right)$$

- Hyperspherical Harmonics

$$\Lambda^2(\Omega) Y_{K,\nu}(\Omega) = -K(K+7) Y_{K,\nu}(\Omega)$$

- Expansion of the wave functions

$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega)$$

- $F = 1$ usual expansion (HH)
- $F = \prod_{i<j} f(r_{ij})$ Correlated expansion (CHH)

HH and CHH method (2)

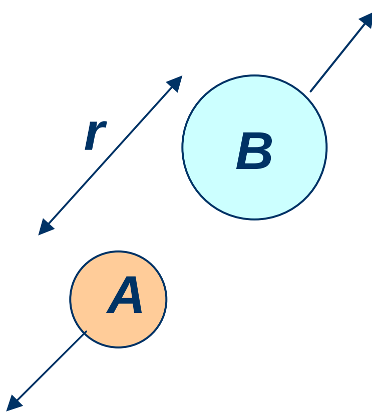
- Bound state

$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega)$$

  - Rayleigh-Ritz variational principle
  - Boundary conditions $u_{K,\mu}(\rho) \to 0 \quad \rho \to \infty$

- Scattering states

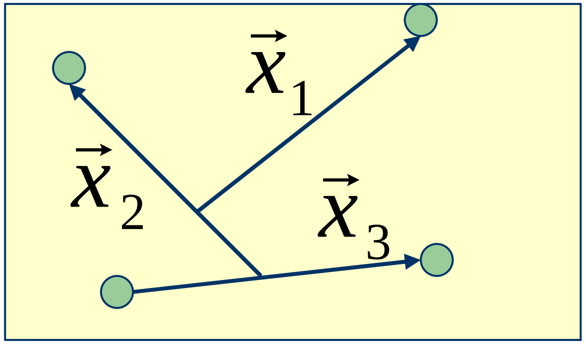
$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega) + \sum_{A,B} \Phi_A \Phi_B \sin(q_{AB} r + \delta_\ell)$$



  - Kohn variational principle
  - Boundary conditions $u_{K,\mu}(\rho) \to \dfrac{e^{iQ\rho}}{\rho^{(3A-1)/2}} \quad \rho \to \infty$

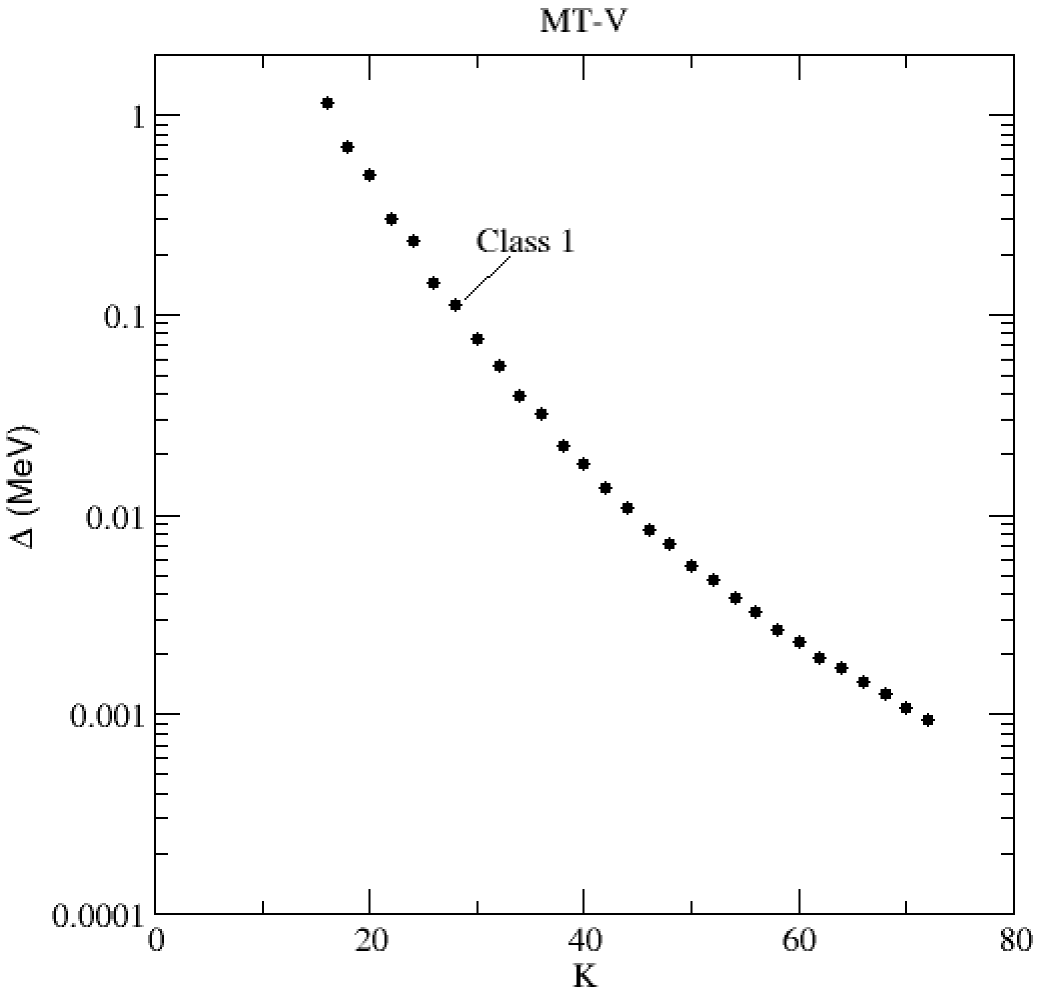
Classes of HH functions (α-particle)

- Class 1: potential basis (PB)
- Class 2: extension of the PB
  - triplet basis TB
- Class 3: HH functions with $\ell_1+\ell_2+\ell_3=2$
- Class 4: HH functions with $\ell_1+\ell_2+\ell_3=4$
- Class 5: HH functions with $\ell_1+\ell_2+\ell_3=6$
- Class 6: HH functions with $T>0$



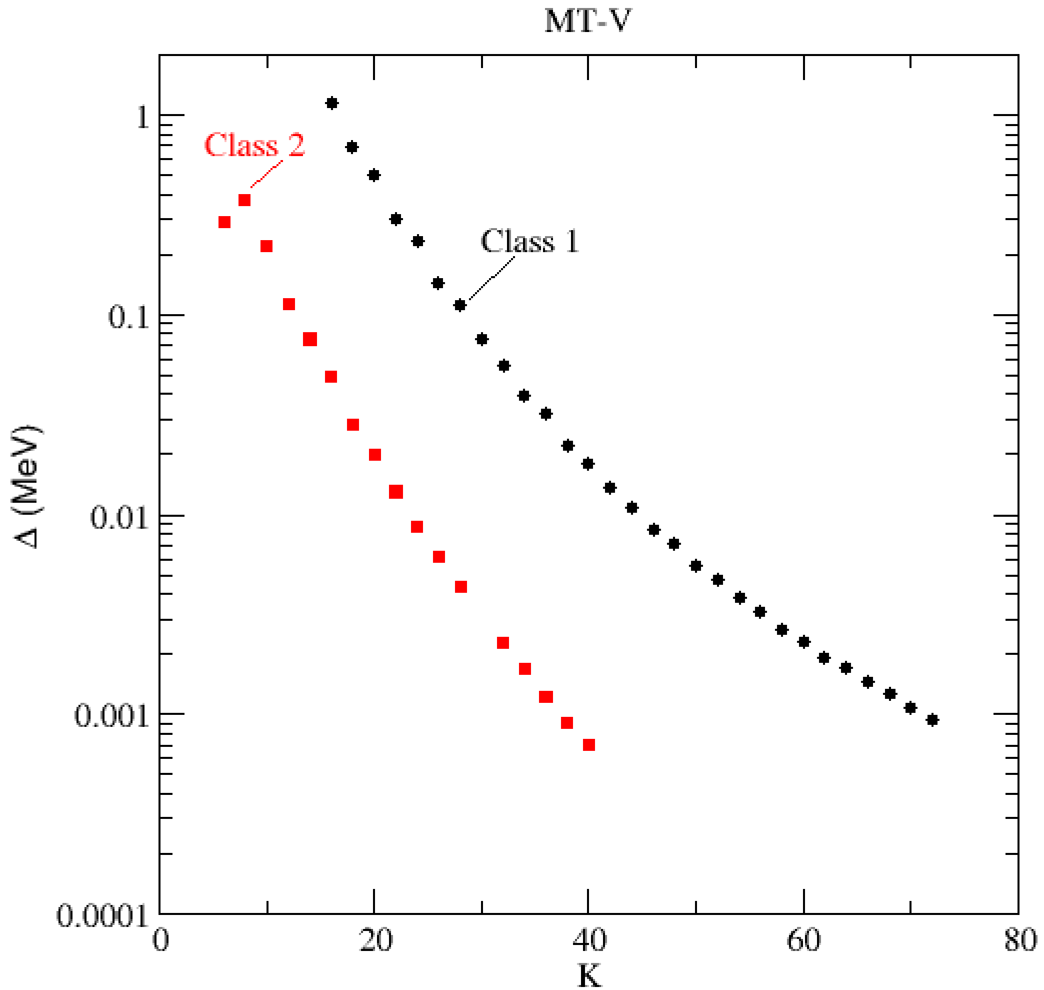
Convergence

$$\Delta = BE(K+2) - BE(K) \approx \frac{b}{K^p}, K \to \infty$$



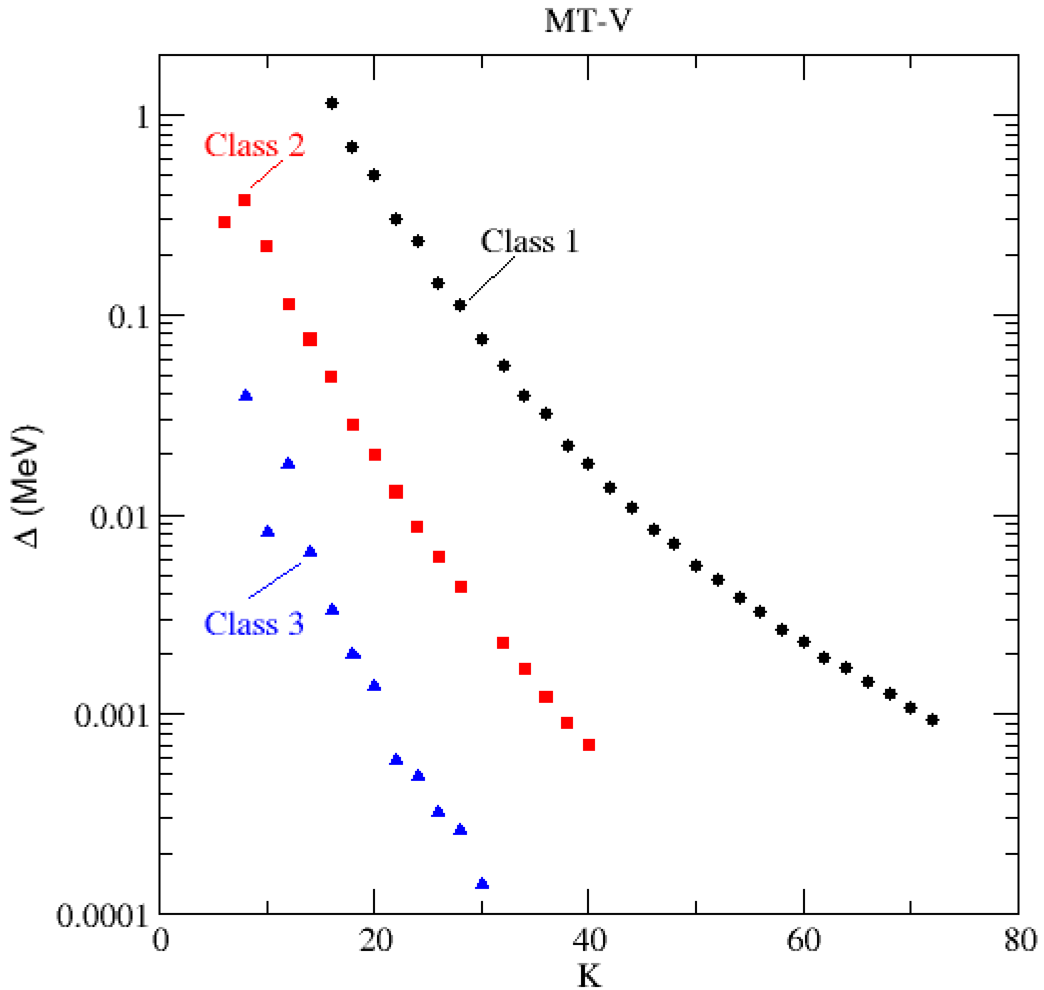
Convergence

$$\Delta = BE(K+2) - BE(K) \approx \frac{b}{K^p}, K \rightarrow \infty$$



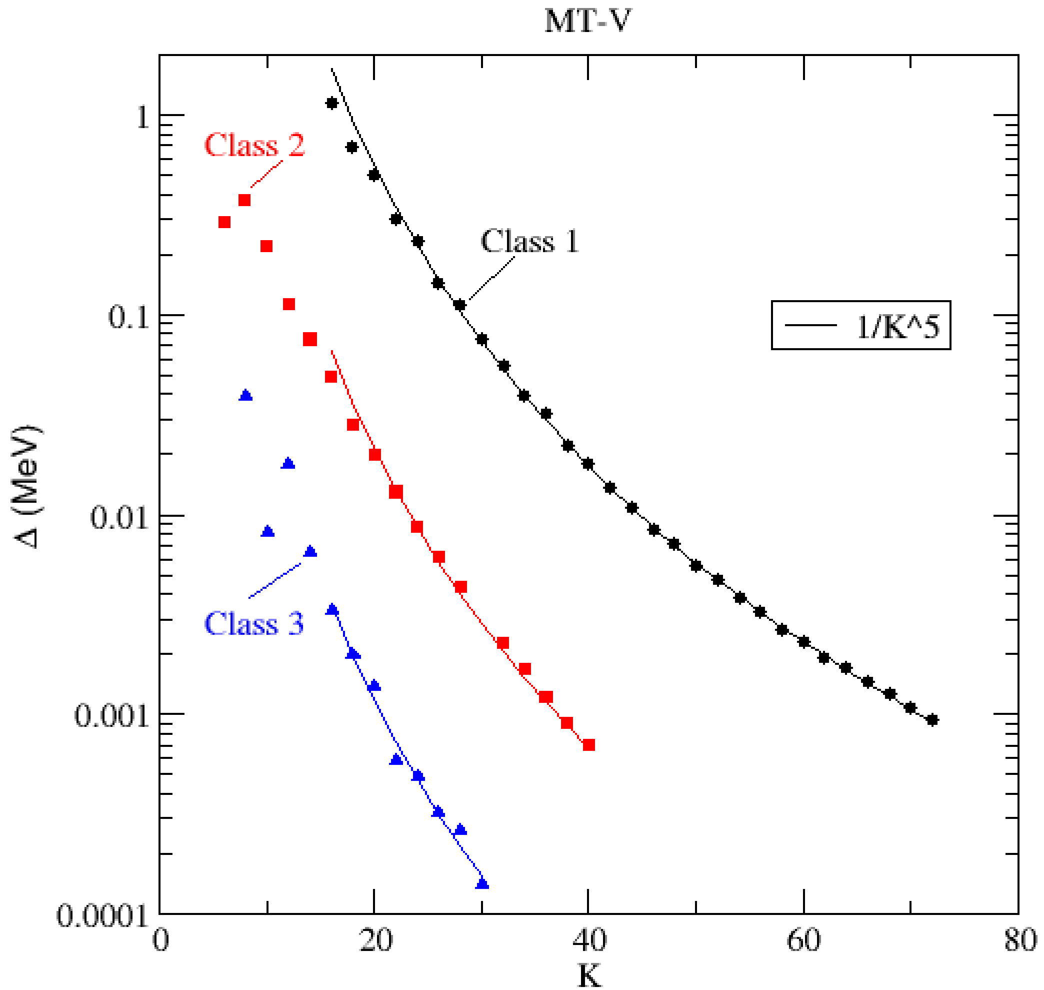
Convergence

$$\Delta = BE(K+2) - BE(K) \approx \frac{b}{K^p}, K \to \infty$$



Convergence

$$\Delta = BE(K+2) - BE(K) \approx \frac{b}{K^{p}}, K \to \infty$$



^3H binding energy

- Binding energy (MeV)

	BE (MeV)			P_D (%)		$P_{T=3/2}$ (%)	
Method	HH	F	NCSM	HH	F	HH	F
AV18	7.618	7.621		8.511	8.510	.002	.002
CD-Bonn	7.998	7.997	7.99	7.02	7.02	.005	.005
N3LO	7.854	7.854	7.85(1)	6.31	6.32	.001	.001

F: Nogga et al, PRC 65, 054003 (2002); Deltuva et al, C68, 024005 (2003)

NCSM: Navratil & Barret, PRC 59, 014311 (2004)

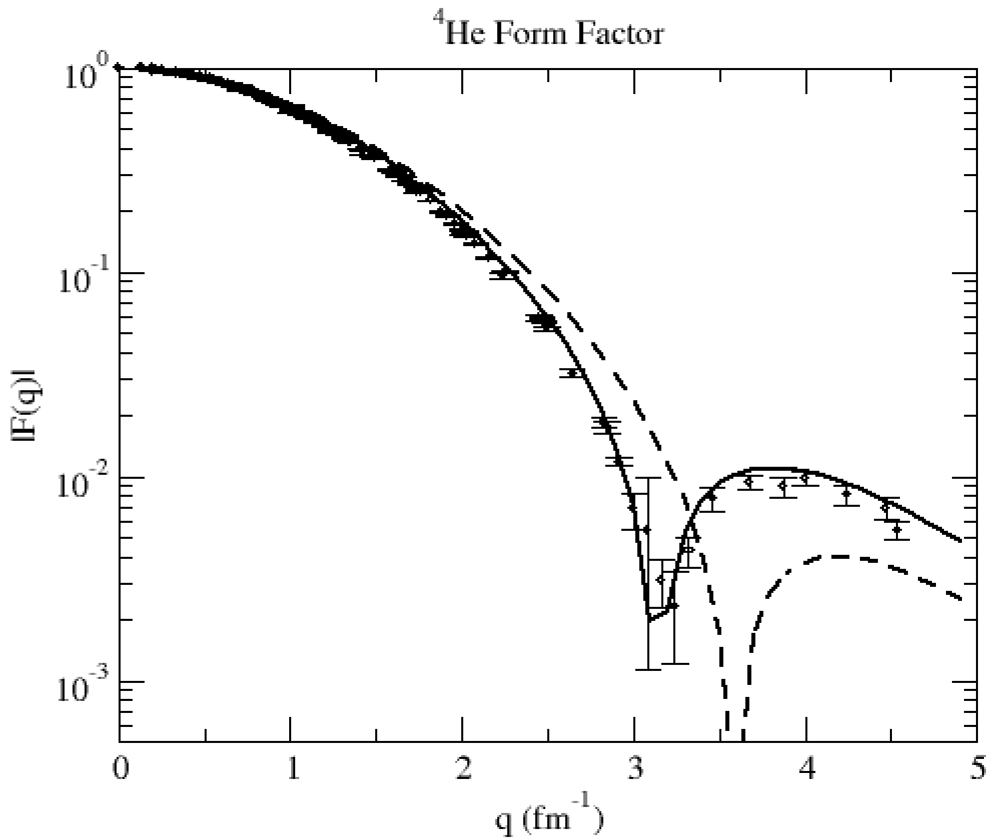
^4He binding energy

	BE (MeV)			P_D (%)		$P_{T>0}$ (%)	
Method	HH	FY	NCSM	HH	FY	HH	FY
AV18	24.22	24.25		13.74	13.78	0.008	0.008
CD-Bonn	26.13	26.16		10.74	10.77	0.014	0.014
N3LO	25.38	25.37	25.36	9.29	9.29	0.006	0.006

FY-Q: Nogga et al, PRC 65, 054003 (2002)

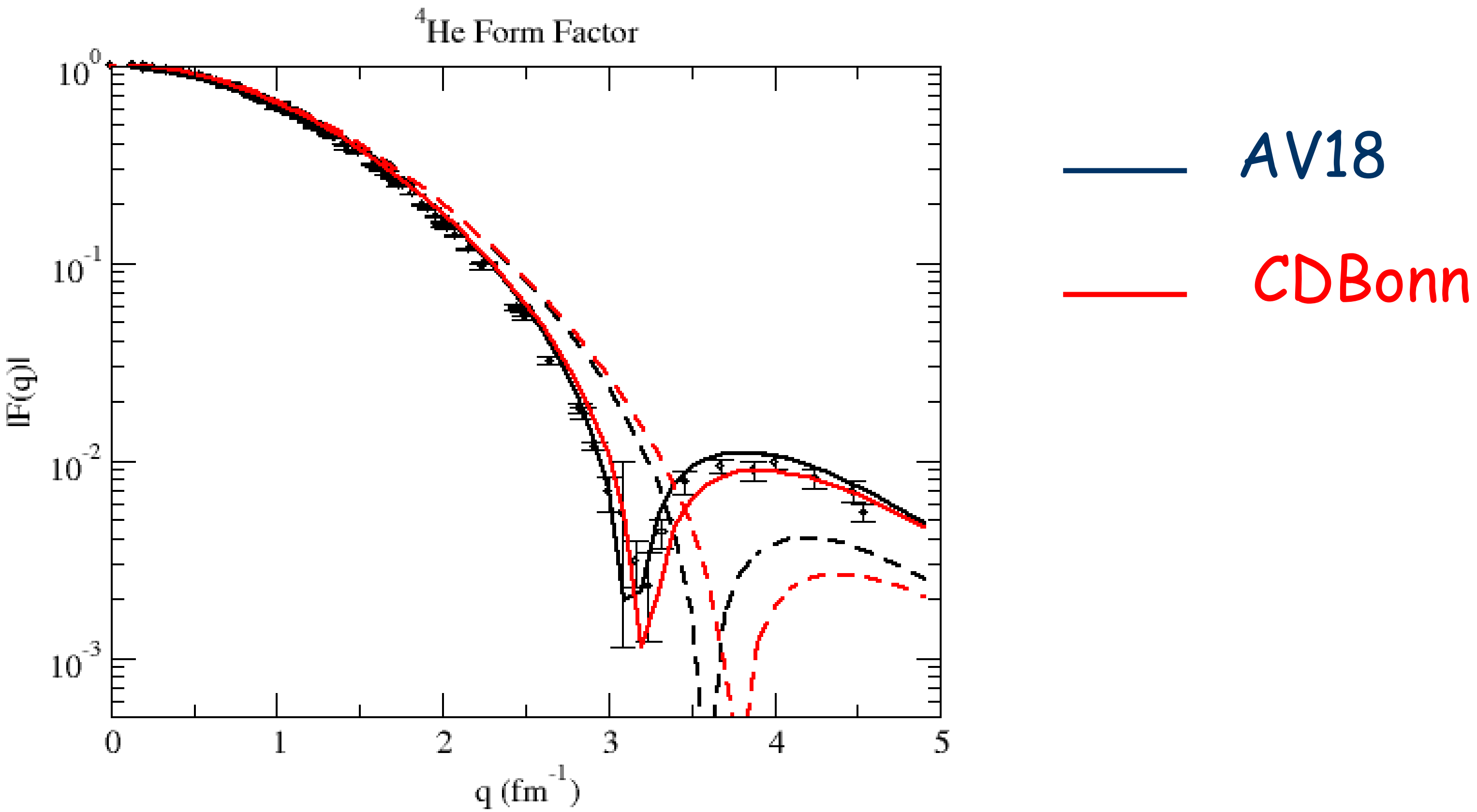
NCSM: Navratil & Barret, PRC 59, 014311 (2004)

^{4}He Form Factor

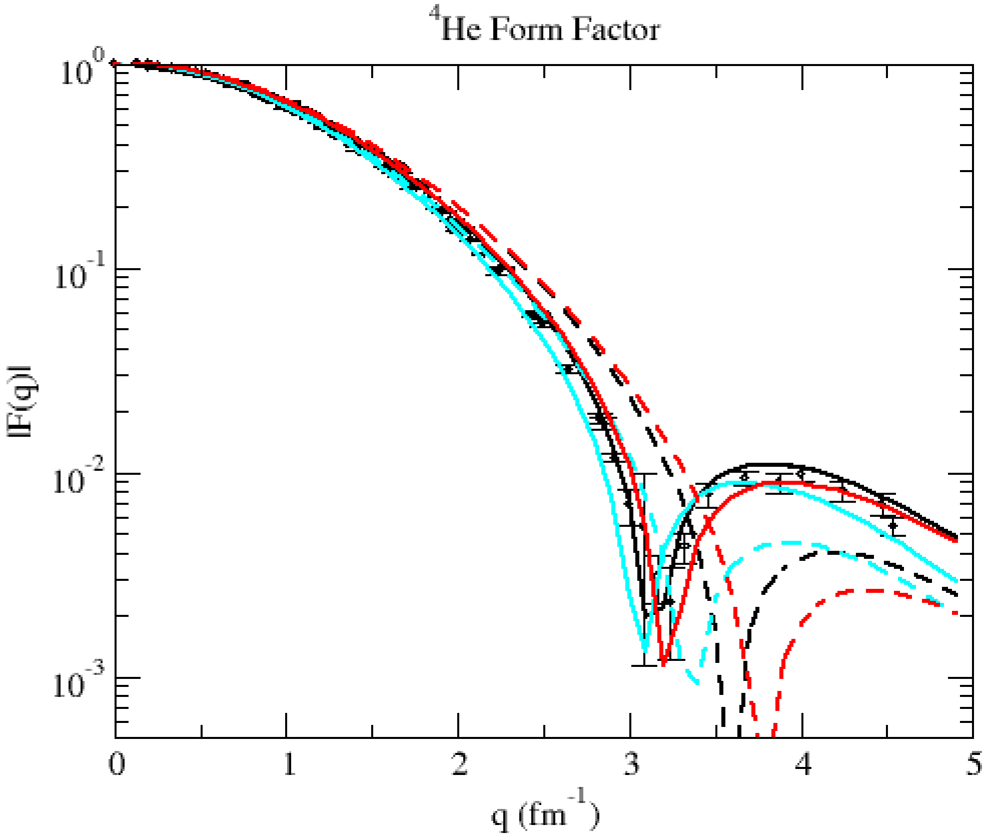


—— AV18

^{4}He Form Factor



^{4}He Form Factor



AV18

CDBonn

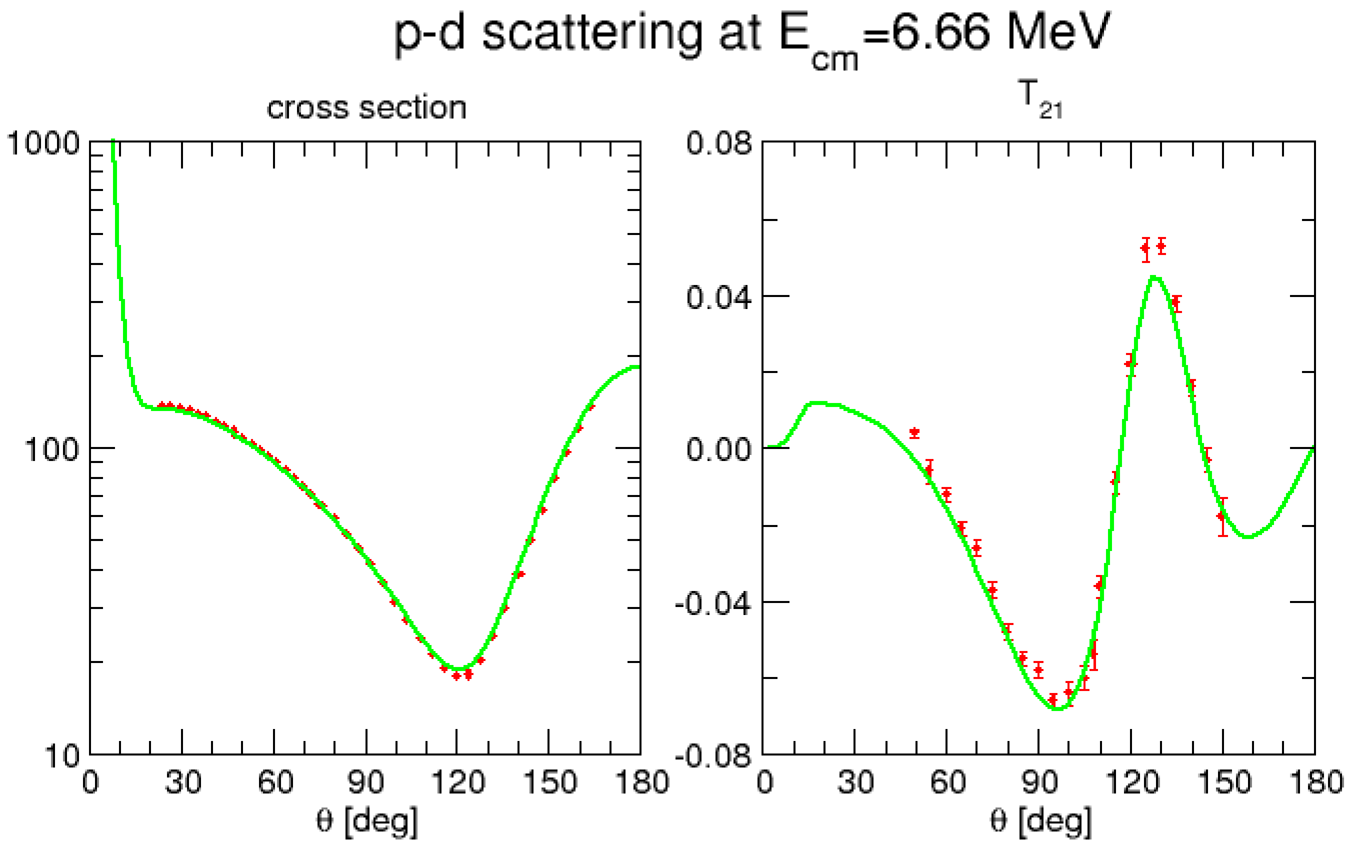
N3LO

A=3 scattering

- N-d scattering
  - Elastic
  - Break-up
- A_y problem
- 3N force effect

Kievsky et al. (2003)

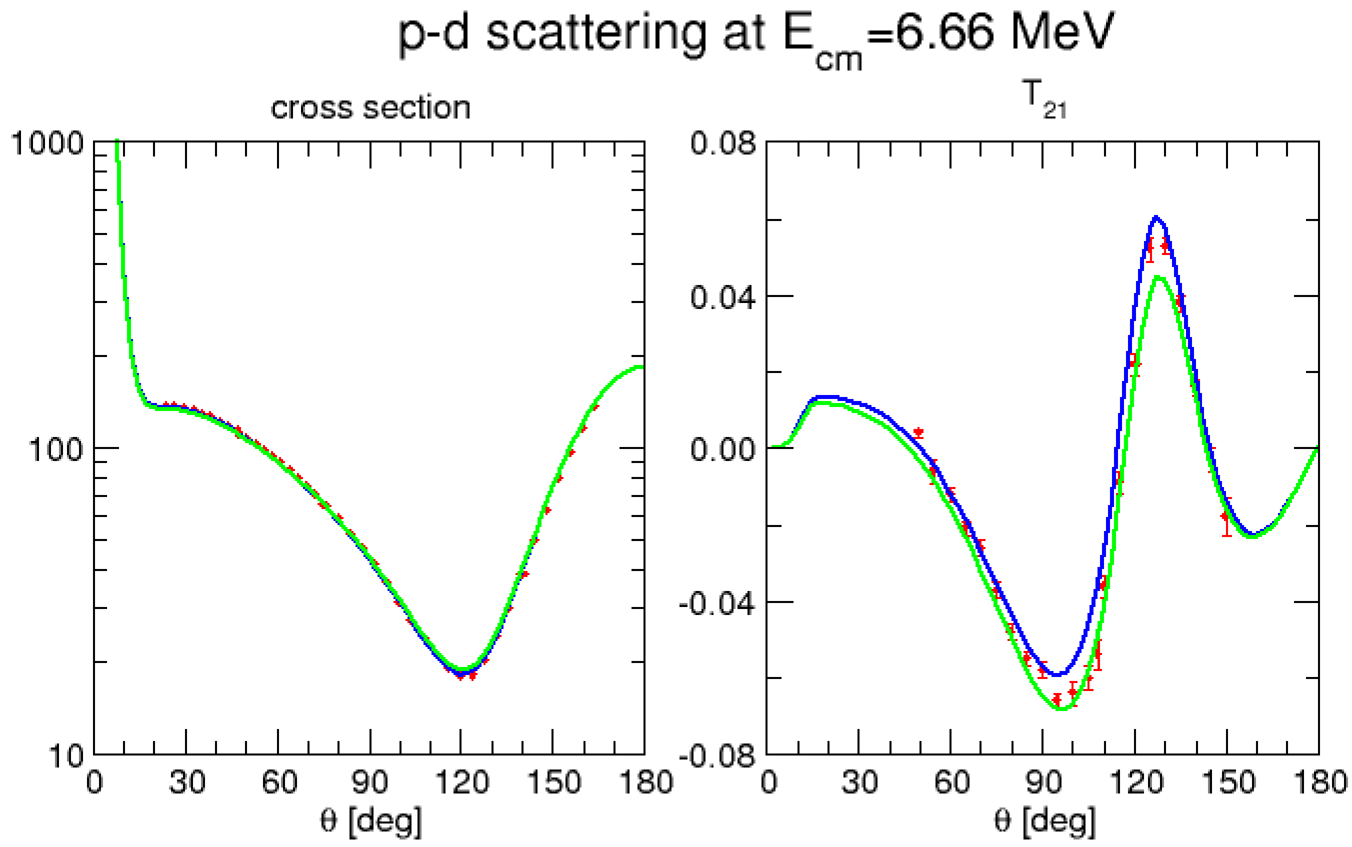
Small effects of the 3N force (1)



NN+C interaction only

Kievsky et al. (2003)

Small effects of the 3N force (1)

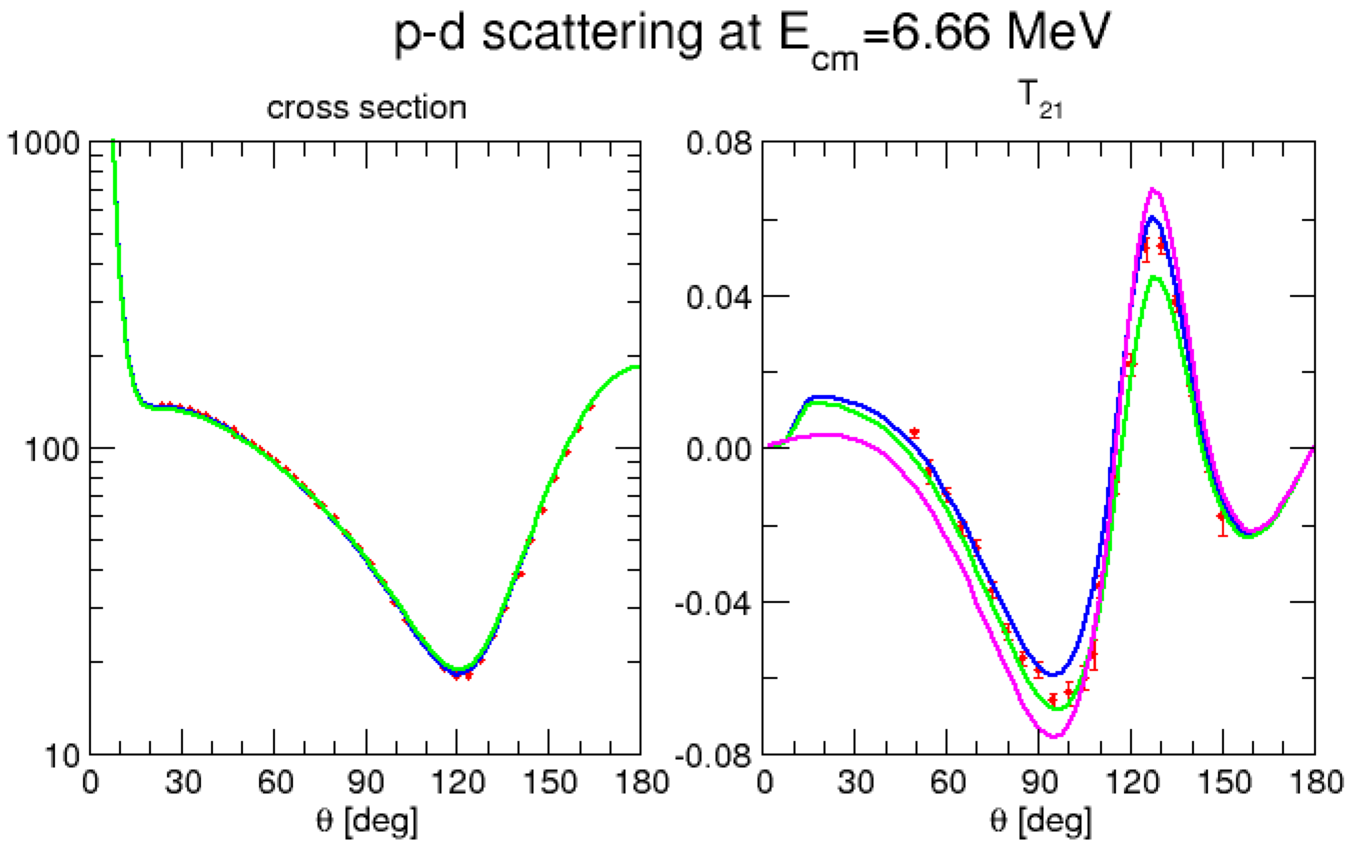


NN+C interaction only

NN+C+NNN interaction

Kievsky et al. (2003)

Small effects of the 3N force (1)



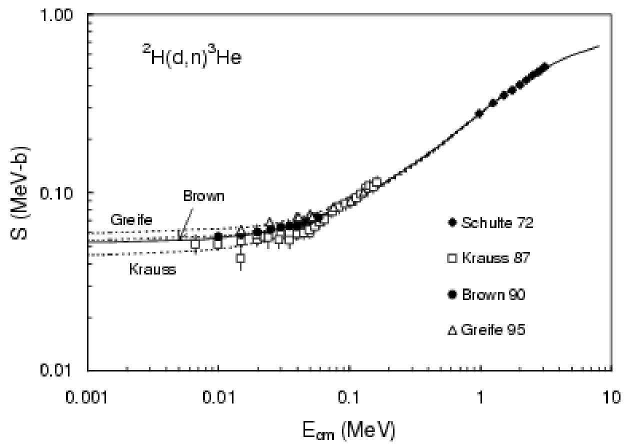
NN+C interaction only

NN+C+NNN interaction

NN interaction only – no Coulomb

A=4 scattering

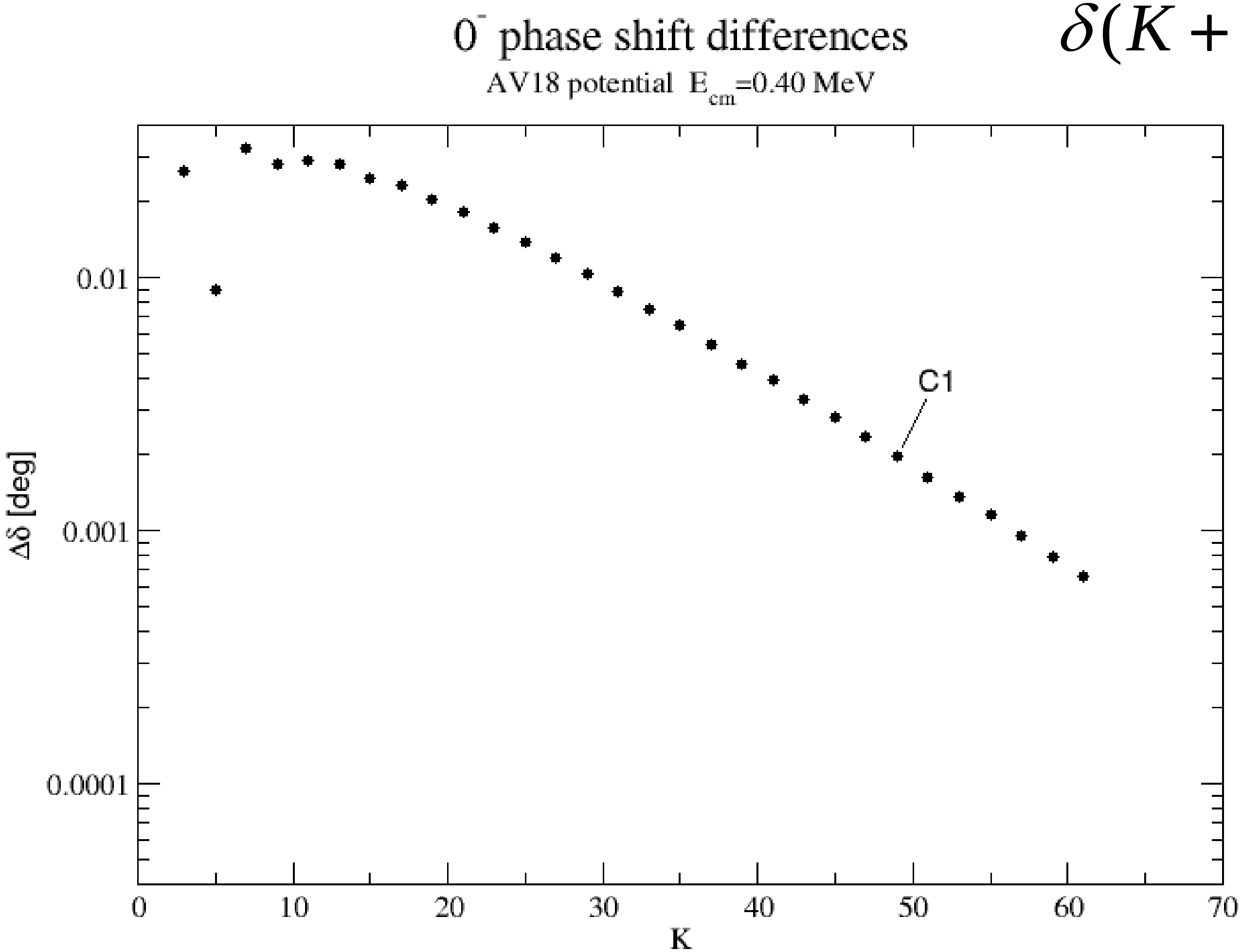
- p-^{3}He, p-^{3}H, d-d,...
  - 3N force effect?
- Fusion



- Application of the HH method under progress for non-local potentials
- Othe methods:
  - FY-R [Lazauskas & Carbonell, 2004]
  - FY-Q (AGS equations) [Fonseca, 1999]
  - Resonating Group Model [Pfitzinger, Hofmann & Hale, 2001]

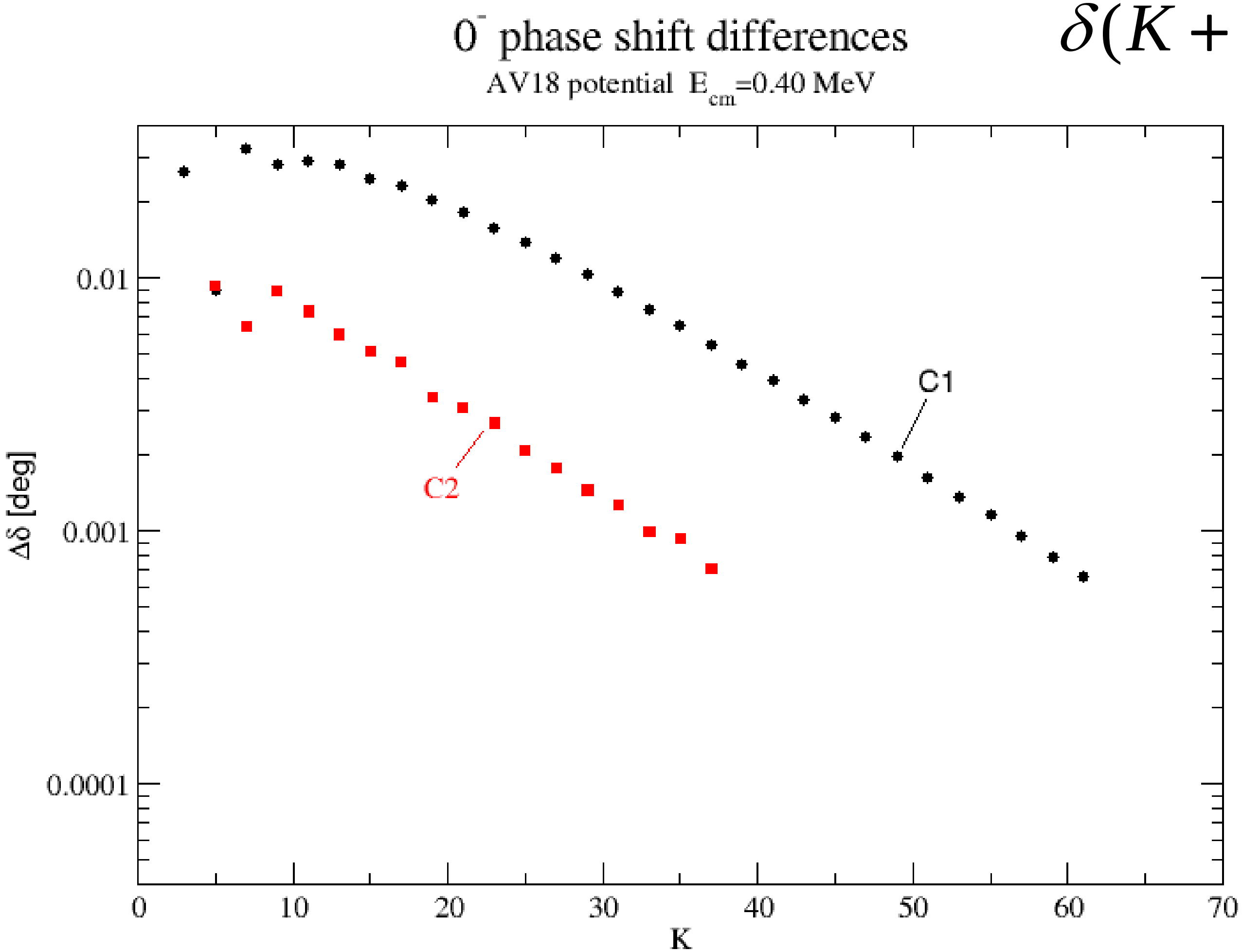
n-^{3}H elastic scattering: convergence

$$\delta(K+2)-\delta(K)$$



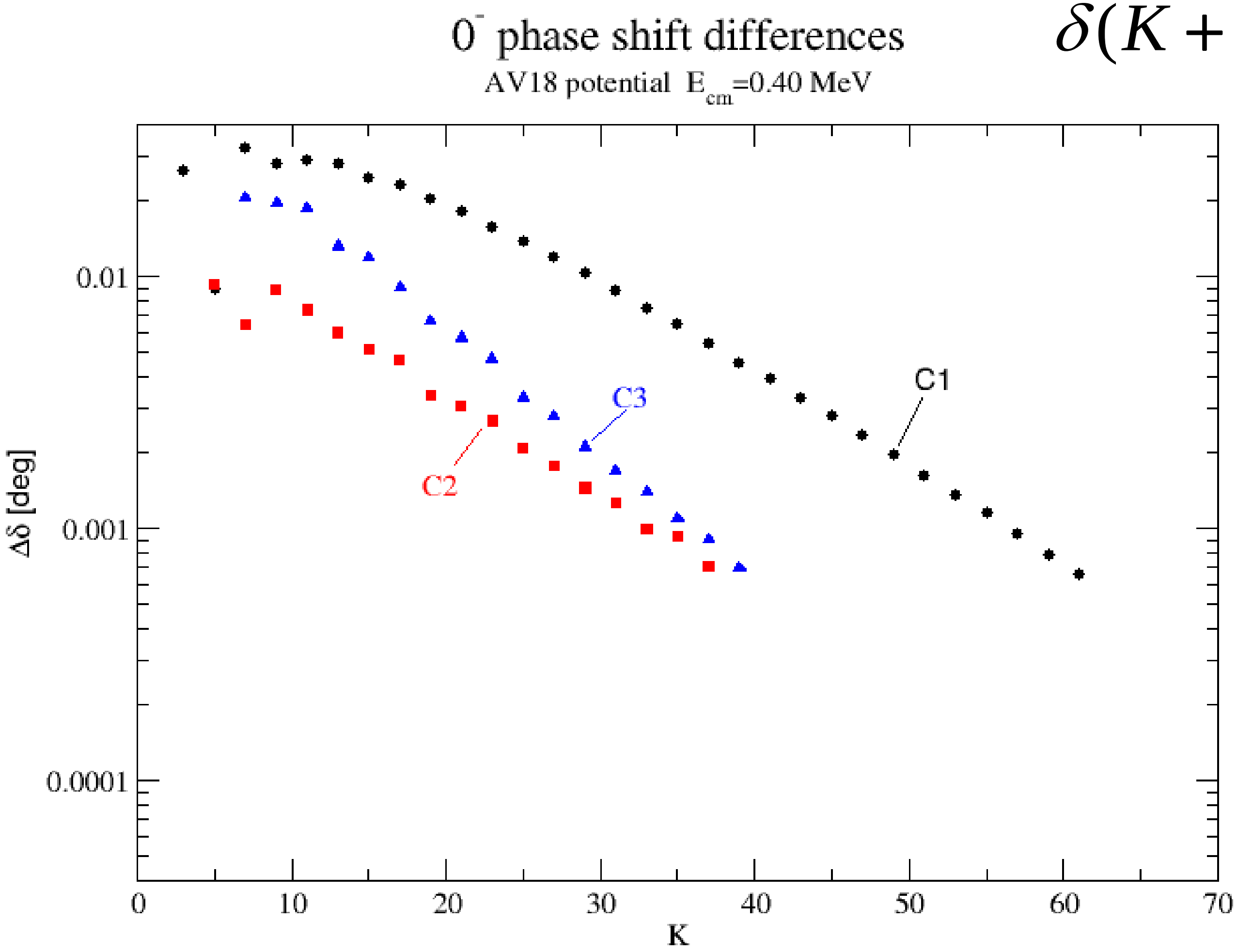
n-^{3}H elastic scattering: convergence

$$\delta(K+2)-\delta(K)$$



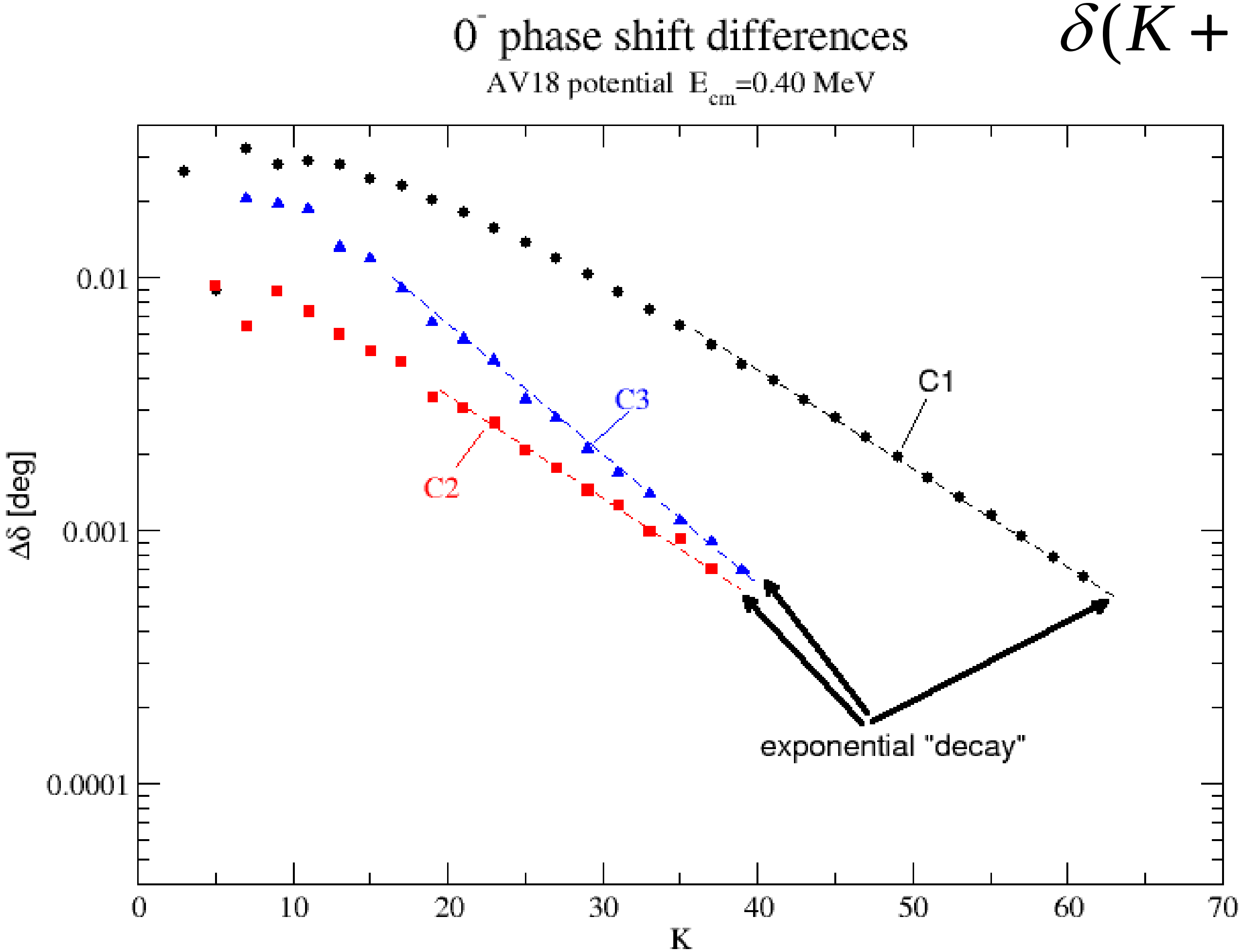
n-^{3}H elastic scattering: convergence

$$\delta(K+2)-\delta(K)$$



n-^{3}H elastic scattering: convergence

$$\delta(K+2)-\delta(K)$$

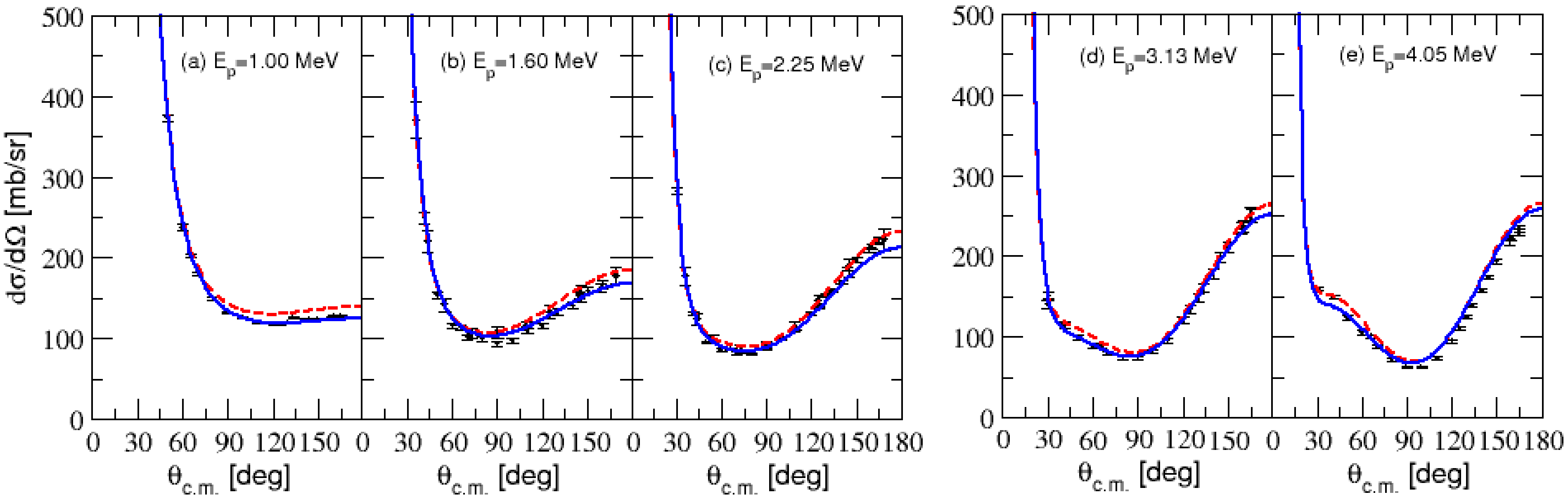


Very small effects from the (standard) 3N forces

p-^{3}He elastic scattering

$$p + {}^3He \rightarrow p + {}^3He$$

Unpolarized cross section



----- AV18

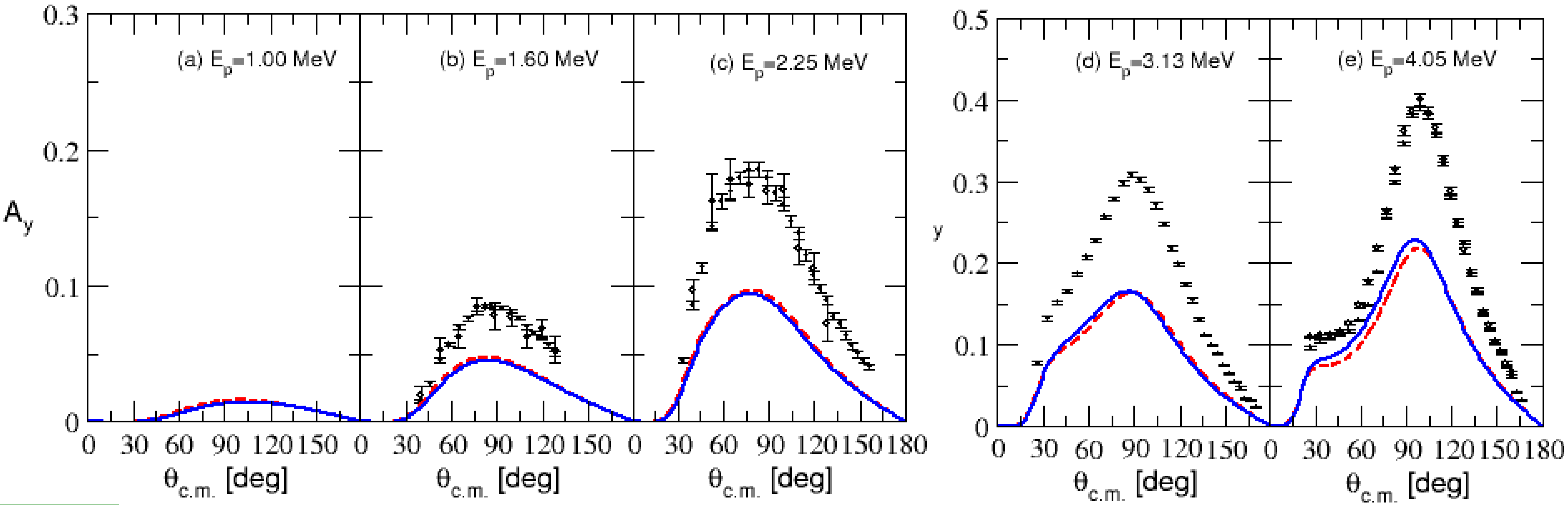
—— AV18+UIX

Very small effects from the (standard) 3N forces

p-^{3}He elastic scattering

$$\vec{p} + {}^3He \rightarrow p + {}^3He$$

Analyzing power



AV18

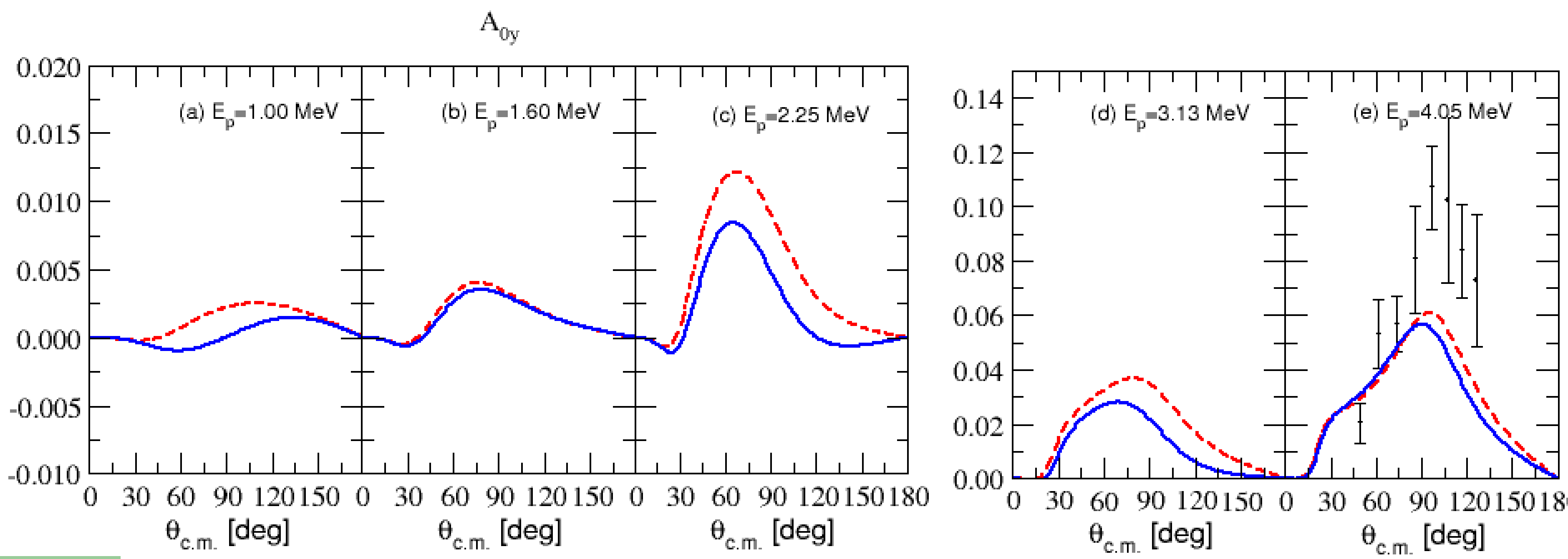
AV18+UIX

Very small effects from the (standard) 3N forces

p-^{3}He elastic scattering

$$p + {}^3\vec{He} \rightarrow p + {}^3He$$

Analyzing power



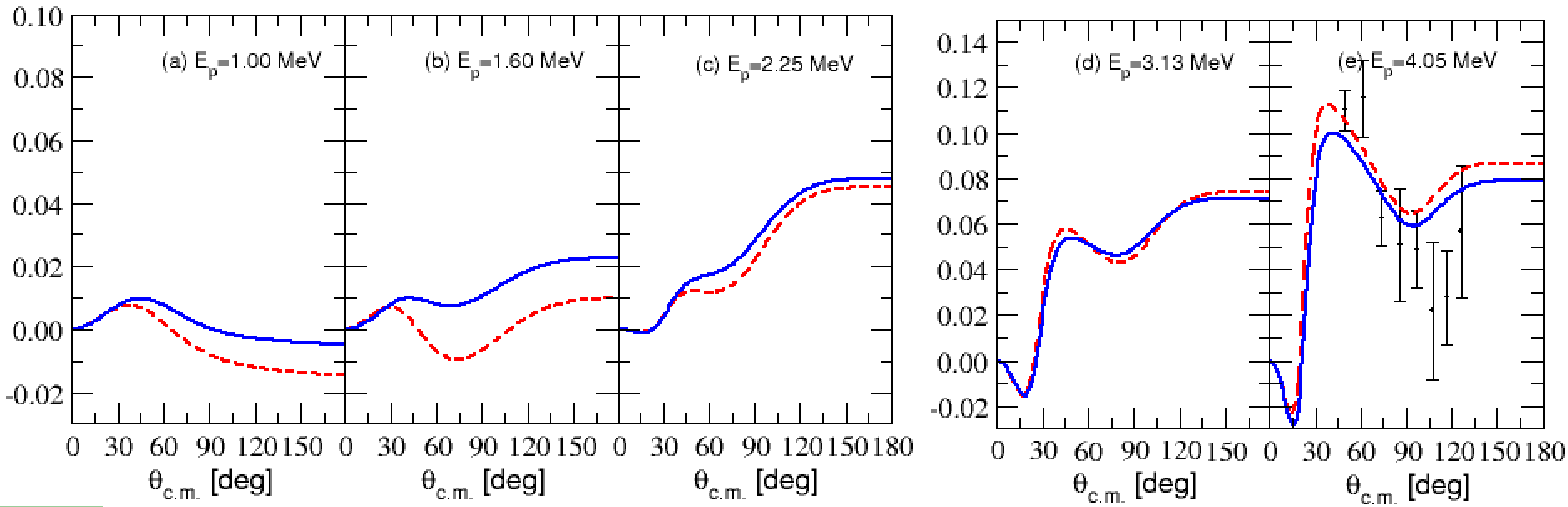
----- AV18

—— AV18+UIX

p-^{3}He elastic scattering

$$\vec{p} + {}^3\vec{He} \rightarrow p + {}^3He$$

Double analyzing power



----- AV18

—— AV18+UIX

Conclusions

- Study of the NN+3N interaction in A=3,4,… systems
- Interest:
  - Contact with QCD (charge symmetry breaking)
    - $d+d \rightarrow {}^4He+\pi^0$
  - Study of reaction of astrophysical interest (solar models, BBN, …)
  - Electron scattering
- Our group: development of the HH method can in momentum space & for non local potentials
  - Scattering with momentum space potentials

HH & CHH method (1)

- Jacobi vectors $\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3$
- Hyperangular variables

$$\rho, \Omega = (\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \phi_2, \phi_3)$$

$$\rho = \sqrt{(\vec{\xi}_1)^2 + (\vec{\xi}_2)^2 + (\vec{\xi}_3)^2}$$

$$\begin{cases} \cos\phi_3 = \xi_3 / \rho \\ \cos\phi_2 = \xi_2 / \sqrt{(\xi_1)^2 + (\xi_2)^2} \end{cases}$$

- Kinetic energy

$$T \to -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial\rho^2} + \frac{8}{\rho}\frac{\partial}{\partial\rho} - \frac{\Lambda^2(\Omega)}{\rho^2}\right)$$

- Hyperspherical Harmonics

$$\Lambda^2(\Omega) Y_{K,\nu}(\Omega) = -K(K+7) Y_{K,\nu}(\Omega)$$

- Expansion of the wave functions

$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega)$$

- ❑ $F = 1$ usual expansion (HH)
- ❑ $F = \prod_{i<j} f(r_{ij})$ Correlated expansion (CHH)

HH and CHH method (2)

- Bound state

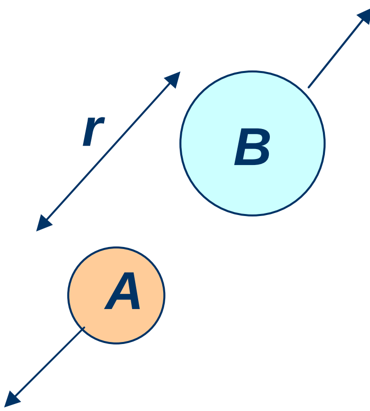
$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega)$$

  - Rayleigh-Ritz variational principle
  - Boundary conditions

$$u_{K,\mu}(\rho) \to 0 \quad \rho \to \infty$$

- Scattering states

$$\Psi = \sum_{K,\mu} F u_{K,\mu}(\rho) Y_{K,\mu}(\Omega) + \sum_{A,B} \Phi_A \Phi_B \sin(q_{AB} r + \delta_\ell)$$



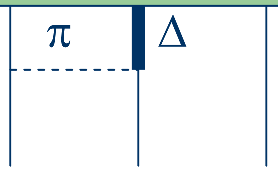
  - Kohn variational principle
  - Boundary conditions

$$u_{K,\mu}(\rho) \to \frac{e^{iQ\rho}}{\rho^{(3A-1)/2}} \quad \rho \to \infty$$

3N interaction

- ${}^3H$, ${}^3He$,… binding energies
- N-d cross section
- Too small splitting of ${}^5He$ states
- Saturation point of nuclear matter
- Etc, etc

- Fujita-Miyazawa [1957]

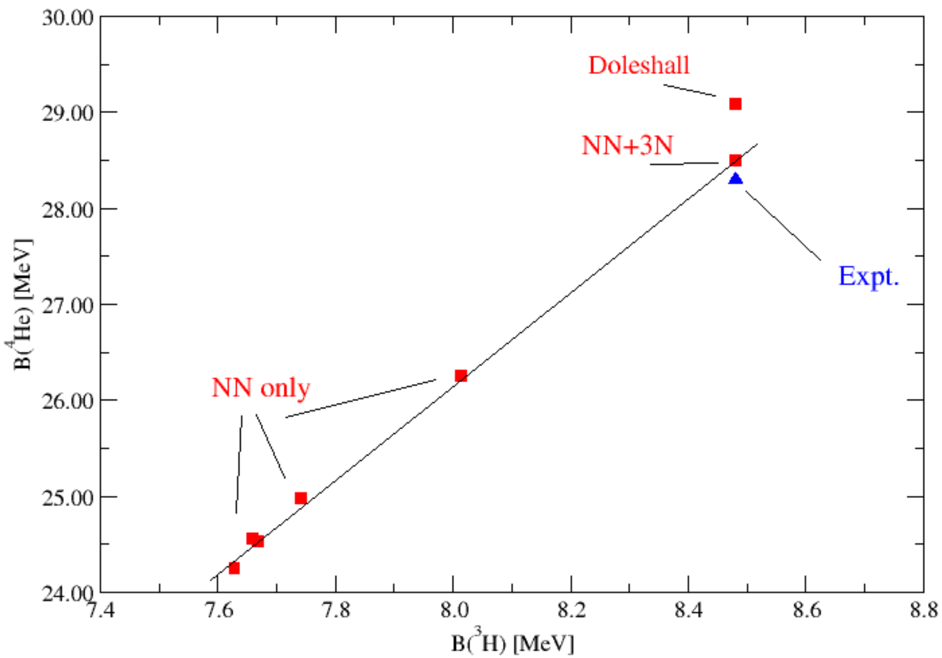


- Commonly used 3N potentials
  - Urbana (UR), Tucson-Melbourne (TM), Brazil (BR)
    - One parameter fitted to reproduce the ${}^3H$ binding energy
- New proposed models
  - ❑ Illinois (3π exchanges) [Pieper et al, 2001]
  - ❑ Chiral symmetry
    - ❖ [Friar et al, 1999]
    - ❖ [Epelbaoum et al, 2002]
  - ❑ π-ρ & ρ-ρ exchanges [Coon & Peňa, 1993]

"Tjon" line

- α-particle vs. ^{3}H binding energy
  - [Nogga et al, 2002, Lazauskas & Carbonell, 2004, MV et al, 2004]

B(^{4}He) (MeV)



B(^{3}H) (MeV)

Dynamics of few-nucleon systems

- Technical methods
  - Faddeev-Yakubovsky Equations
  - AGS equations
  - Green Function Monte Carlo (GFMC)
  - Variational methods
    - Gaussian basis
    - Hyperspherical Harmonics (HH)
    - RGM
  - Effective methods
    - No-core Harmonic Oscillator method
    - EIHH
  - [See, for example, Carlson & Schiavilla, RMP **70**, 743 (1998)]
- 3N bound and scattering state problems can be now (almost) "routinely" solved
  - [See, for example, Gloeckle et al, Phys. Rep. **274**, 107 (1996)]
- Accurate study of e.m. and weak transition

Further developments

- Effective methods

Finite basis *P*

$$H\psi = E\psi \quad \Rightarrow$$

$$H_P\psi(P) = E_P\psi(P)$$

E_p converges to E very rapidly

▪ *HO basis* [Navratil & Barrett (1999)]

▪ *HH basis* [Barnea et al., (2000)]

- Relativity
  - How much is important?
- Retardation in pion exchange is claimed to explain the A_y puzzle in N-d scattering
  - [Canton & Shadow, 2000]
- Relativistic dynamics
  - Instant form
  - Front form
  - Point form
  - [Keister & Polizou, 1991,…]

HH method: 3NF

- Truncation of the 3NF

$$\tilde{W}(i,j,k) = |P_K^+\rangle W(i,j,k)\langle P_K|$$

$$\langle P_K | HH_{K'}\rangle = 0 \text{ if } K' > K$$

K	B (MeV)	<T> (MeV)
20	27.351	109.71
24	27.366	109.67
30	27.372	109.65
34	27.374	109.64

AV18+UIX

Test calculation C1-C3

See also:

Barnea et al, nucl-th/0404086

$$\Lambda = \ell_1 + \ell_2 + \ell_3$$

Contribution of the different classes

- AV18 potential

Class	BE (MeV)
PB	16.12
PB+TB	20.03
Λ=2	23.39
Λ= 4	24.17
Λ= 6	24.19
T>0	24.21

- AV18+UR potential

Class	BE (MeV)
PB	16.38
PB+TB	20.92
Λ=2	27.37
Λ= 4	28.41
Λ= 6	28.44
T>0	28.46

Extrapolation using the $(1/K)^p$ law: $\Delta B \approx 0.01$ MeV

(*) Extrapolated

^{4}He binding energy - benchmark 2

- AV18 potential

Method	BE (MeV)	<T> (MeV)
HH (*)	24.22	97.84
FY-R	24.22	
FY-Q	24.23	97.80

- AV18+UR potential

Method	BE (MeV)	<T> (MeV)
HH (*)	28.47	113.30
FY-Q	28.50	113.21
GFMC	28.34(4)	

FY-R: Lazauskas & Carbonell (2004)

FY-Q: Nogga et al, PRC 65, 054003 (2002)

GFMC: Wiringa et al, PRC 62, 014001 (2000)

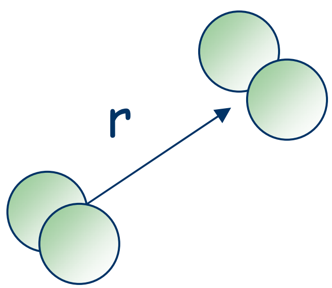
^{4}He asymptotic normalization constants

- d-d separation

$$\beta = \sqrt{2m(B_4 - 2B_2)}$$

$$\Psi \rightarrow \sum_{L=0,2} a_L \left\{ \left[\Phi_d \Phi_d \right]_L Y_L(\hat{r}) \right\}_{00} \frac{e^{-\beta r}}{r}$$

$$D_2 \approx \frac{a_2}{\beta^2 a_0}$$



Extracted from
(α,dd) experiments

[Weller et al, 1986]

	D_2
HH (AV18+UR)	-0.11
Adhikari et al. 1994	-0.12
Expt.	-0.2±0.05

FY-Q calculation: Nogga et al, PRC **65**, 054003 (2002)

Isopin *T>0* components in the ^{4}He w.f.

- Energy gain 20 keV

AV18 potential		
Method	$P_{T=1}$ (%)	$P_{T=2}$ (%)
HH	0.0028	0.0052
FY-Q	0.003	0.005

Pot.	$P_{T=1}$ (%)	$P_{T=2}$ (%)
AV18	0.0028	0.0052
NIJ-II	0.0016	0.0074
AV18+ UIX	0.0025	0.0050

- Origin of the mixing

Hamilt.	$10^3 x P_{T=1}$ (%)	$10^3 x P_{T=2}$ (%)
H_0	0	0
+Coul	1.5	0.1
+CSB	3.0	4.9
+e.m. +Δm	2.8	5.2