# Breakup of halo nuclei within a Dynamical Eikonal Model

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# Outline

- Introduction on halo nuclei
- Dynamical eikonal description of the breakup of halo nuclei
- Application: Breakup of $^{11}$Be on $^{12}$C and $^{208}$Pb
- Conclusion

# Introduction: Halo Nuclei

- Characterised by a very large matter radius
- Exhibit a low separation energy of 1 or 2 neutrons

⇒strongly clusterised system:
neutrons tunnel far from the core and form a halo
Examples:

| Nucleus | $S_n$ or $S_{2n}$ |
|---|---|
| $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$ | 0.504 MeV |
| $^{11}\mathrm{Li} \equiv {}^{9}\mathrm{Li} + 2\mathrm{n}$ | 0.300 MeV |

# Breakup reaction

Halo nuclei are short-lived ⇒studied in indirect ways
Breakup = dissociation of the core + halo structure
by interaction with a target

⇒Need accurate theoretical description of breakup
coupled to realistic model of projectile

Various breakup models exist:

- Semiclassical: $P$-$T$ motion $\equiv$ classical trajectory (perturbation theory, eikonal, time-dependent)
- Quantum: fully quantum (DWBA, CDCC)

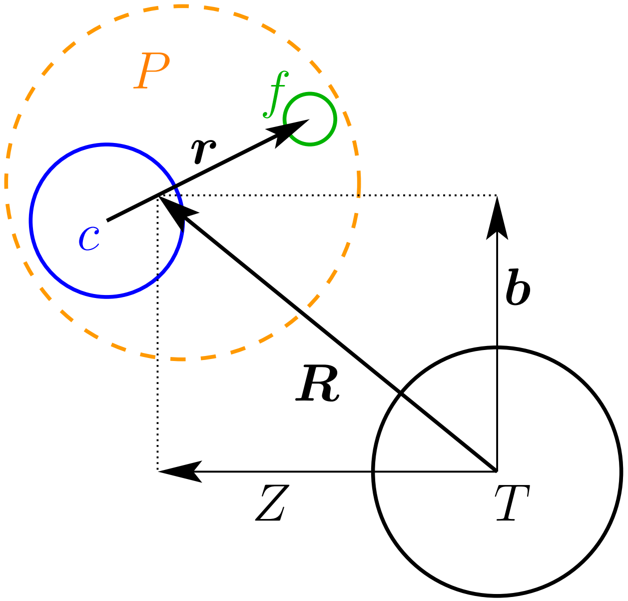
New model: Dynamical Eikonal Approximation
unifies eikonal and time-dependent

D. Baye, P.C., G. Goldstein, PRL 95, 082502 (2005)
G. Goldstein, D. Baye, P.C., PRC 73, 024602 (2006)

# Inputs

Projectile ($P$) modelled as a two-body system:
core ($c$)+loosely bound nucleon ($f$) described by

$H_0 = T_r + V_{cf}(\boldsymbol{r})$




$V_{cf}$ adjusted to reproduce
bound states and
some resonances

Target $T$ seen as structureless particle

$P$-$T$ interaction simulated by optical potentials

$\Rightarrow$breakup reduces to three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{R},\boldsymbol{r}) = E_T\Psi(\boldsymbol{R},\boldsymbol{r})$$

# Dynamical eikonal approximation (1)

Three-body scattering problem

$$[T_R + H_0 + V_{cT} + V_{fT}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0(\boldsymbol{r})$

To suppress the rapid variation in $\boldsymbol{R}$ we factorise

$\Psi(\boldsymbol{r},\boldsymbol{R}) = e^{iKZ}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{R})$:

$$\begin{aligned} H\Psi &= e^{iKZ}[T_R + vP_Z + \frac{1}{2}\mu_{PT}v^2 \\ &\quad + (H_0 + V_{cT} + V_{fT})]\widehat{\Psi} \end{aligned}$$

Neglecting $T_R$ vs $vP_Z$ and using $E_T = \frac{1}{2}\mu_{PT}v^2 + E_0$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

# Dynamical eikonal approximation (2)

$$i\hbar v \frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_0 + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

is equivalent to a TDSE with straight line trajectories

BUT here $\boldsymbol{b}$ and $Z$ are quantal $\Rightarrow$no trajectory

The usual eikonal uses adiabatic approx. $H_0 - E_0 \sim 0$

$\Rightarrow$ neglects internal dynamical effects of projectile

$$\widehat{\Psi}^{\text{eik}}(\boldsymbol{r},\boldsymbol{b},Z) = e^{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ'\left[V_{cT}(\boldsymbol{r},\boldsymbol{b},Z')+V_{fT}(\boldsymbol{r},\boldsymbol{b},Z')\right]}\Phi_0(\boldsymbol{r})$$

$\Rightarrow$ dynamical eikonal generalises TDSE and eikonal

- improves TDSE by including interferences
- improves eikonal by including dynamical effects
- we know how to solve accurately TDSE

# Breakup cross section

Breakup transition matrix element:

$$T_{fi}^{\mathrm{bu}} = \langle e^{i\boldsymbol{K'}\cdot\boldsymbol{R}}\chi_{\boldsymbol{k}}^{(-)}|V_{cT}+V_{fT}|e^{iKZ}\widehat{\Psi}\rangle,$$

where $H_0\chi_{\boldsymbol{k}}^{(-)} = E\chi_{\boldsymbol{k}}^{(-)}$ (ingoing scattering wave)

$$\begin{aligned}
T_{fi}^{\mathrm{bu}} &= \langle e^{i\boldsymbol{K'}\cdot\boldsymbol{R}}\chi_{\boldsymbol{k}}^{(-)}|H_0 - E + V_{cT} + V_{fT}|e^{iKZ}\widehat{\Psi}\rangle \\
&= \langle e^{i\boldsymbol{K'}\cdot\boldsymbol{R}}\chi_{\boldsymbol{k}}^{(-)}|i\hbar v\frac{\partial}{\partial Z} + E_0 - E|e^{iKZ}\widehat{\Psi}\rangle \\
&\approx i\hbar v\int d\boldsymbol{b}e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\int_{-\infty}^{\infty} dZ\frac{\partial}{\partial Z}\langle\chi_{\boldsymbol{k}}^{(-)}|\widehat{\Psi}\rangle,
\end{aligned}$$

assuming $\boldsymbol{q} = \boldsymbol{K'} - K\widehat{\boldsymbol{Z}}$ transverse and $E - E_0 \sim 0$

$$T_{fi}^{\mathrm{bu}} = i\hbar v\int d\boldsymbol{b}e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\langle\chi_{\boldsymbol{k}}^{(-)}|\widehat{\Psi}(Z\to\infty)\rangle$$

# Breakup cross section (2)

After integration over $\phi_b$,

$$T_{fi}^{\text{bu}} \propto \sum_{lm} Y_l^m(\Omega_k) e^{i(m_0-m)\varphi} \int_0^\infty J_{|m_0-m|}(qb) S_{klm}(b) b db,$$

where $S_{klm}(b) = \langle \Phi_{klm} | \widehat{\Psi}(Z \to \infty) \rangle$
contains all breakup information

Cross sections:

$$\frac{d\sigma_{\text{bu}}}{d\boldsymbol{k} d\Omega} \propto |T_{fi}^{\text{bu}}|^2 \xrightarrow{\int d\Omega_k, \int d\Omega} \frac{d\sigma_{\text{bu}}}{dE d\Omega} \text{ and } \frac{d\sigma_{\text{bu}}}{dE}$$

$\Rightarrow$Dynamical eikonal extends TDSE
to new observables (differential cross sections)
taking into account interferences between *trajectories*

# $^{11}$Be

We illustrate the technique for $^{11}$Be on C and Pb
$^{11}$Be is the best known one-neutron halo nucleus

Its breakup on C and Pb measured at RIKEN
Fukuda *et al.* PRC 70, 054606 (2004)

$^{10}$Be-n potential chosen to reproduce first three states

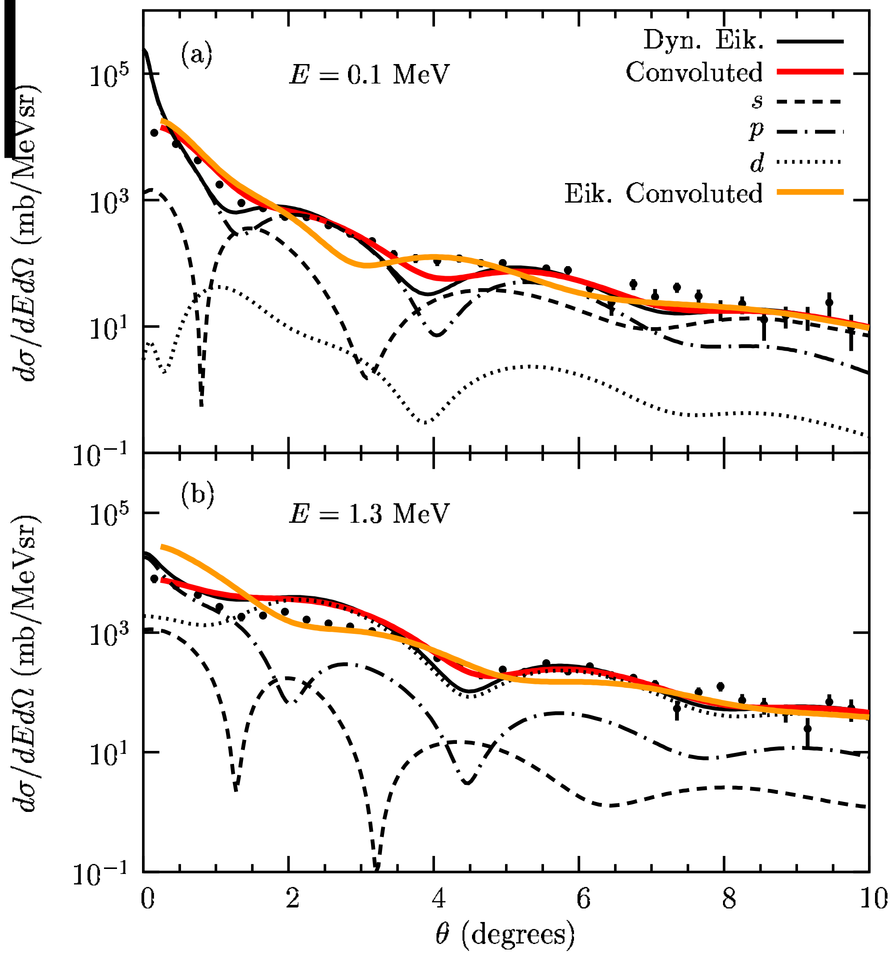
| | | |
|---|---|---|
| $5/2^+$ | 1.274 | $d5/2$ |
| $1/2^-$ | -0.184 | $0p1/2$ |
| $1/2^+$ | -0.504 | $1s1/2$ |

We use a W-S with parity dependence depth plus S-O

Optical potentials $V_{cT}$ and $V_{fT}$ chosen in the literature

⇒All parameters adjusted *a priori*: not fitting param.

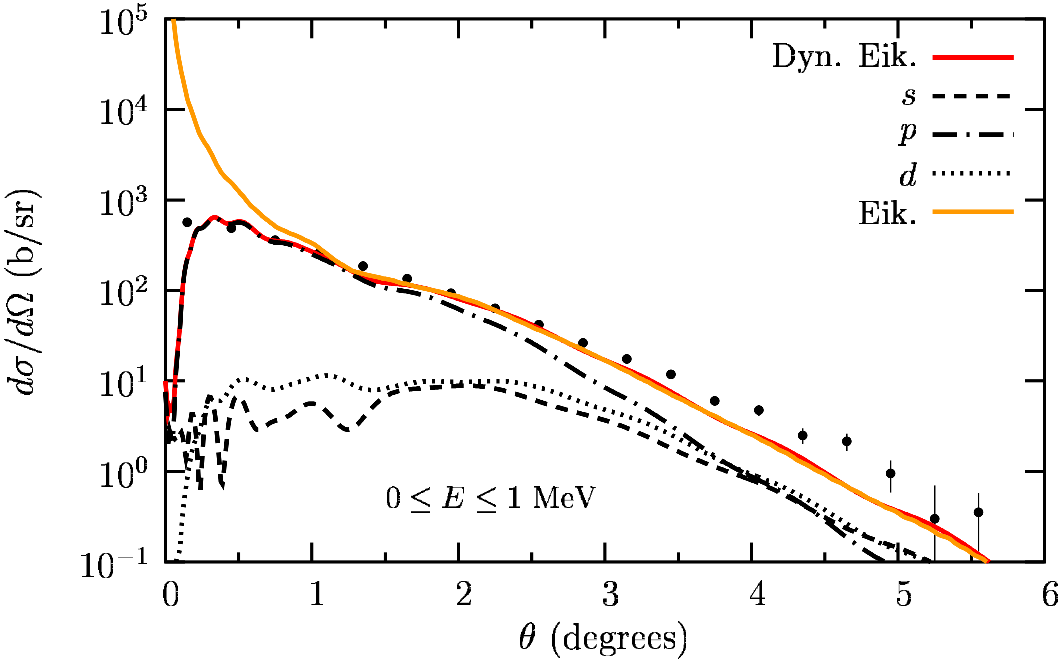
# Breakup of $^{11}$Be on $^{12}$C @ $67A$MeV




- Good agreement with experiment (no param.)
- Diffraction pattern in all partial waves
- Eikonal: different pattern
- Dominance: $p$ at 0.1MeV; $d$ at 1.3MeV=$E_{\rm res}$

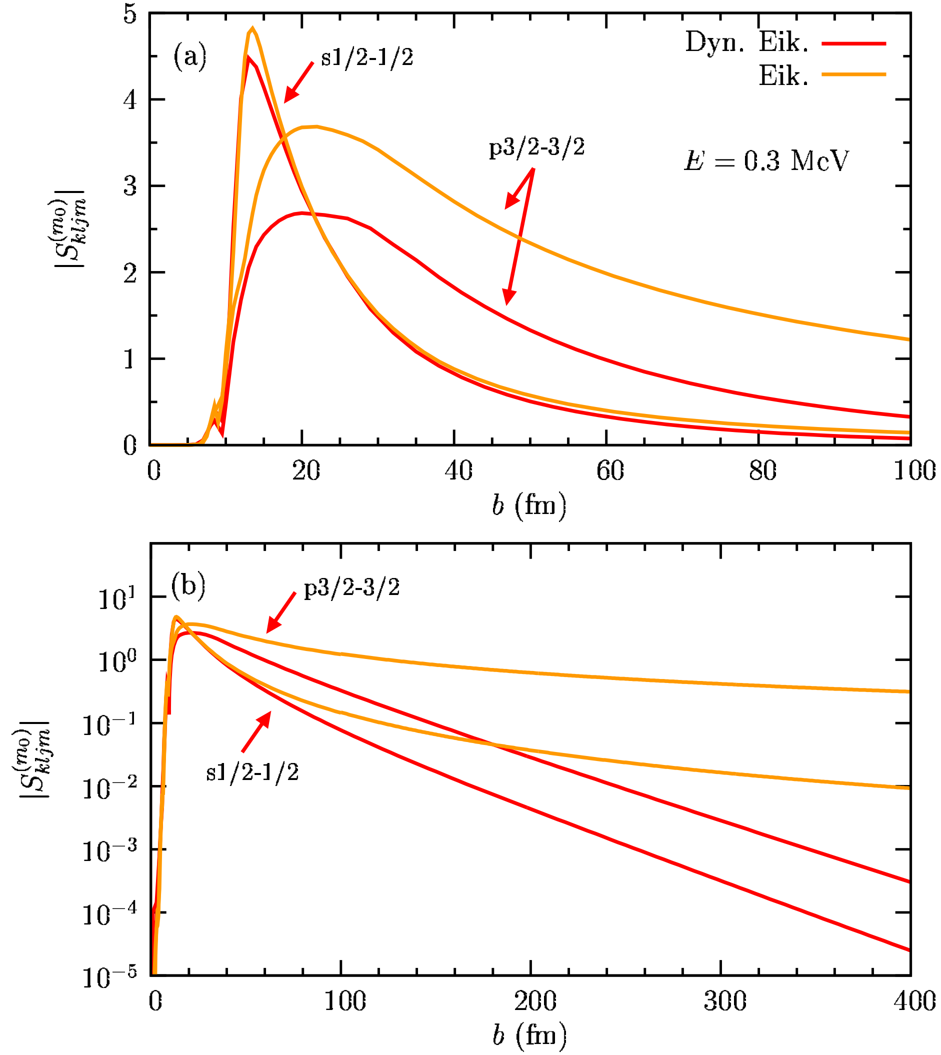
Dynamical eikonal extends TDSE to light targets and differential observables

# Breakup of $^{11}$Be on Pb @ $69A$MeV




- Good agreement with experiment (no parameter)
- $p$ waves at small $\theta$, where Coulomb dominates
- All waves at large $\theta$, where nuclear dominates (theory slightly underestimates data)
- Eikonal diverges at $\theta = 0°$, but agrees with dynamical at large $\theta$

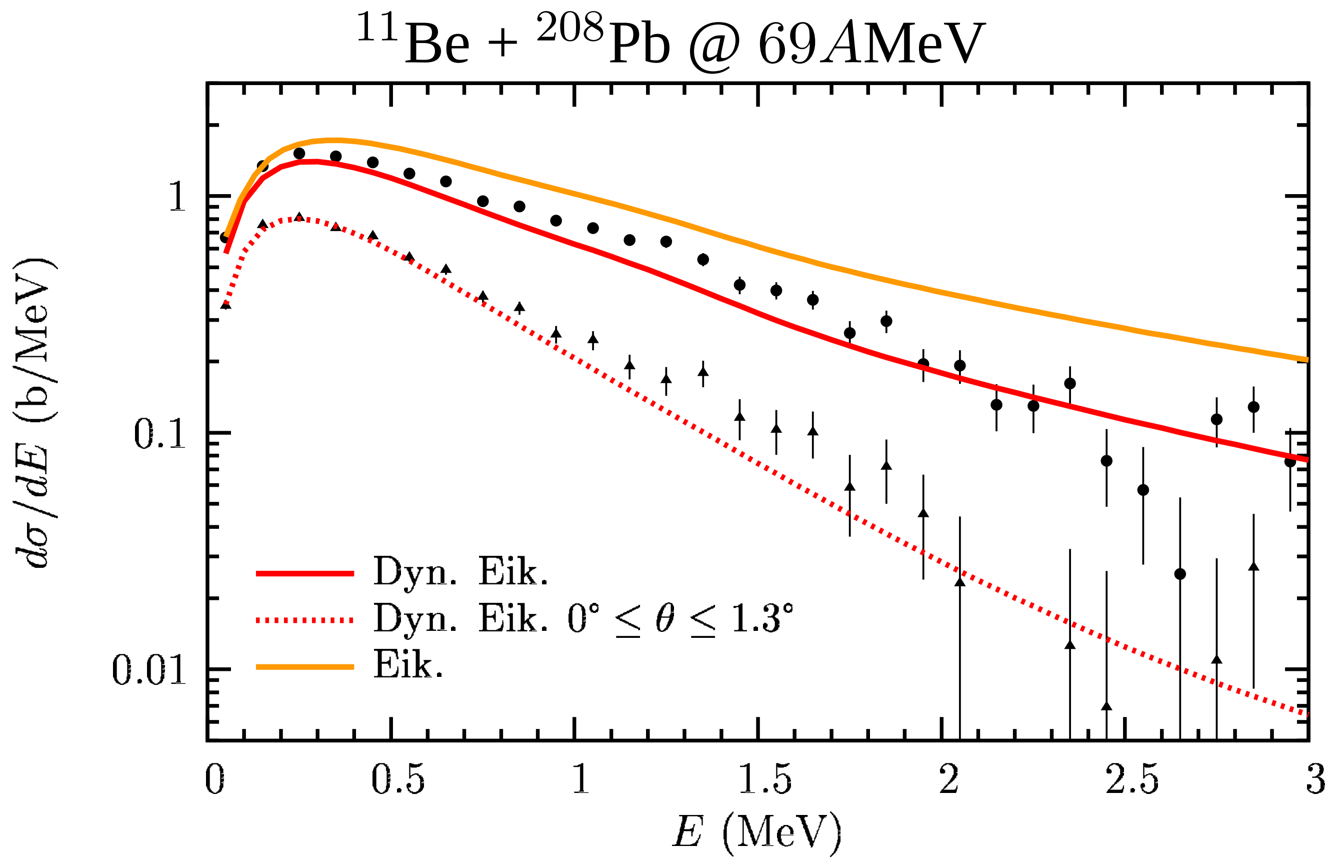
# Comparison with eikonal




Eikonal not right asymptotics ($1/b$ whereas dyn. $\propto e^{-ab}$)

$\Rightarrow$diverges when Coulomb dominates (small $\theta$)

# Relative energy distribution




- Good agreement with experiment
  OK for $\theta \leq 1.3°$ where Coulomb dominates
  underestimates data when all $\theta$ included (nuclear?)
- Eikonal diverges $\Rightarrow$ needs a cutoff at large $b$

# Conclusion

- Breakup is a tool to study halo nuclei
- Dynamical Eikonal Approximation
  is a new reaction model
  unifies Time-Dependent and Eikonal:
  - includes interferences between trajectories
  - includes internal dynamics of the projectile

  ⇒extends TDSE to differential observables
  Goldstein, Baye, P.C., PRC 73, 024602 (2006)
- Good agreement with experiment
  for both light and heavy targets

⇒One model for all targets and cross sections
at intermediate energies

Future: improve the projectile description

# Elastic scattering

Transition matrix elements reads

$$T_{fi}^{\text{el}} = \langle e^{i\boldsymbol{K'}\cdot\boldsymbol{R}}\Phi_0|V_{cT}+V_{fT}|e^{iKZ}\widehat{\Psi}\rangle$$

$$= \int d\boldsymbol{R}e^{i(KZ-\boldsymbol{K'}\cdot\boldsymbol{R})}\langle\Phi_0|i\hbar v\frac{\partial}{\partial Z} - H_0 + E_0|\widehat{\Psi}\rangle$$

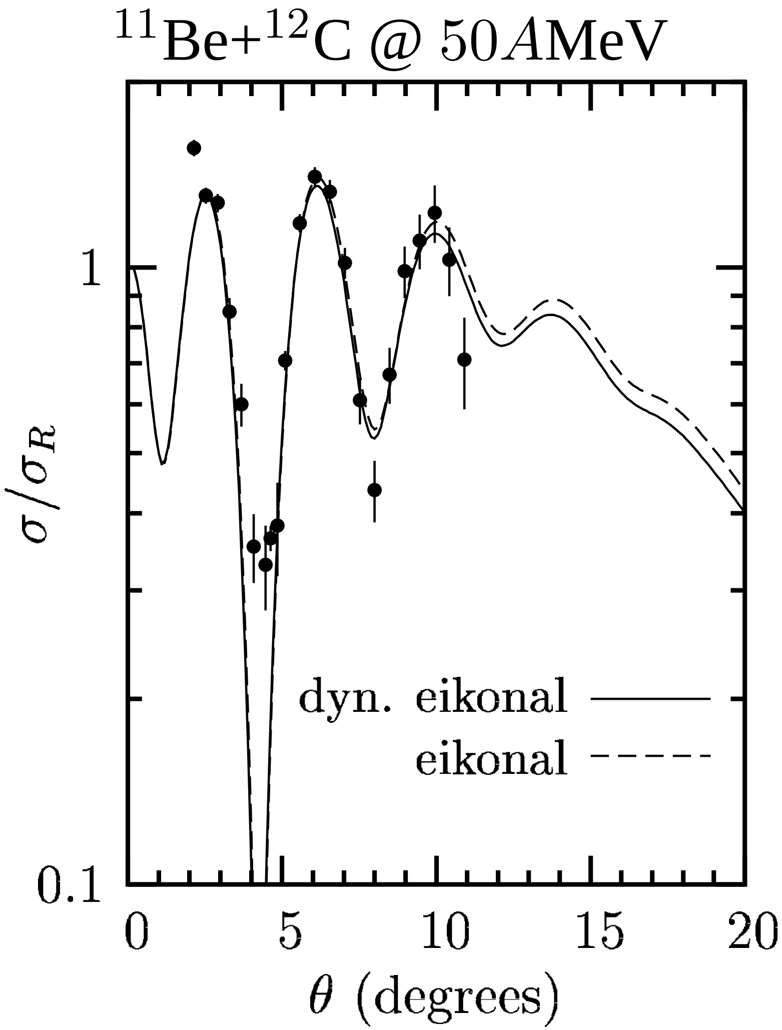
Setting $S_0(b) = \langle\Phi_0|\widehat{\Psi}(b, Z\to\infty)\rangle - 1$ gives

$$T_{fi}^{\text{el}} \approx i\hbar v\int d\boldsymbol{b}e^{i\boldsymbol{q}\cdot\boldsymbol{b}}S_0(b) = 2i\pi\hbar v\int_0^\infty J_0(qb)S_0(b)bdb$$

Using $q = 2K\sin\theta/2$, we get $\frac{d\sigma_{\text{el}}}{d\Omega} \propto |T_{fi}^{\text{el}}|^2$

- eikonal approx. is obtained replacing $\widehat{\Psi} \longrightarrow \widehat{\Psi}^{\text{eik}}$
- TDSE angular distributions use of $b_{\text{cl.}} = \frac{2\eta}{K\theta}$
  $\Rightarrow$ no interference

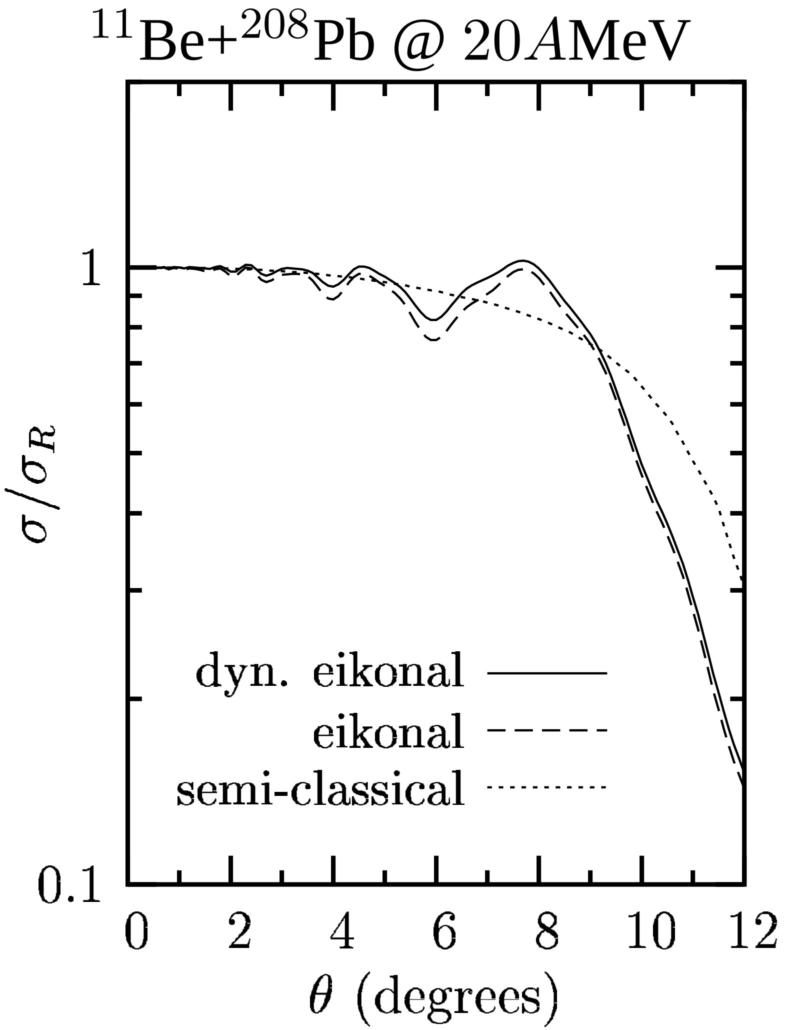
# Elastic scattering of $^{11}$Be on $^{12}$C




- Good agreement with experiment (No fitting parameter)
- Large interferences $\Rightarrow$TDSE cannot be used
- Small difference between dynamical and usual eikonal

$\Rightarrow$ little dynamical effects in elastic scattering (on light target?)

# Elastic scattering of $^{11}$Be on $^{208}$Pb




- Smaller interferences
  TDSE fairly valid at
  forward angles
- Small difference between
  dyn. and usual eikonal
  ⇒ little dynamical effects
  in elastic scattering

Validates usual eikonal for describing elastic scattering

# Eikonal and time-dependent models

Eikonal: scattering wave function approximated by

$$\Psi^{\mathrm{eik}} = \exp\left[-i\int_{-\infty}^{Z} dZ' V_{PT}(\boldsymbol{b} + Z'\widehat{\boldsymbol{Z}})\right]\Phi_0$$

$+$ simple approximation with all degrees of freedom

$-$ no internal dynamics of the projectile

$-$ no good asymptotic behaviour

Time-dependent: $P$-$T$ motion = classical trajectories

$\Rightarrow$ wave function solution of TDSE:

$$i\frac{\partial \Psi^{\mathrm{TDSE}}}{\partial t} = \{H_0 + V_{PT}[R(t)]\}\,\Psi^{\mathrm{TDSE}}$$

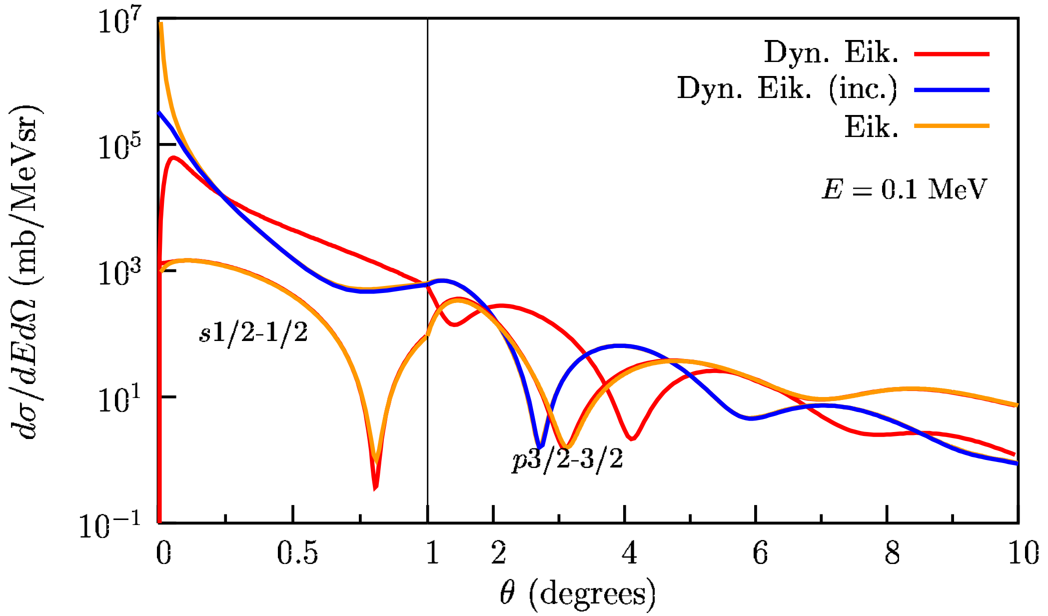
$+$ includes dynamics inside the projectile

$-$ classical trajectories $\Rightarrow$ no interference effects

# Coherent and incoherent rotations

Two ways of rotating $\widehat{\Psi}$ along $\phi_b$:

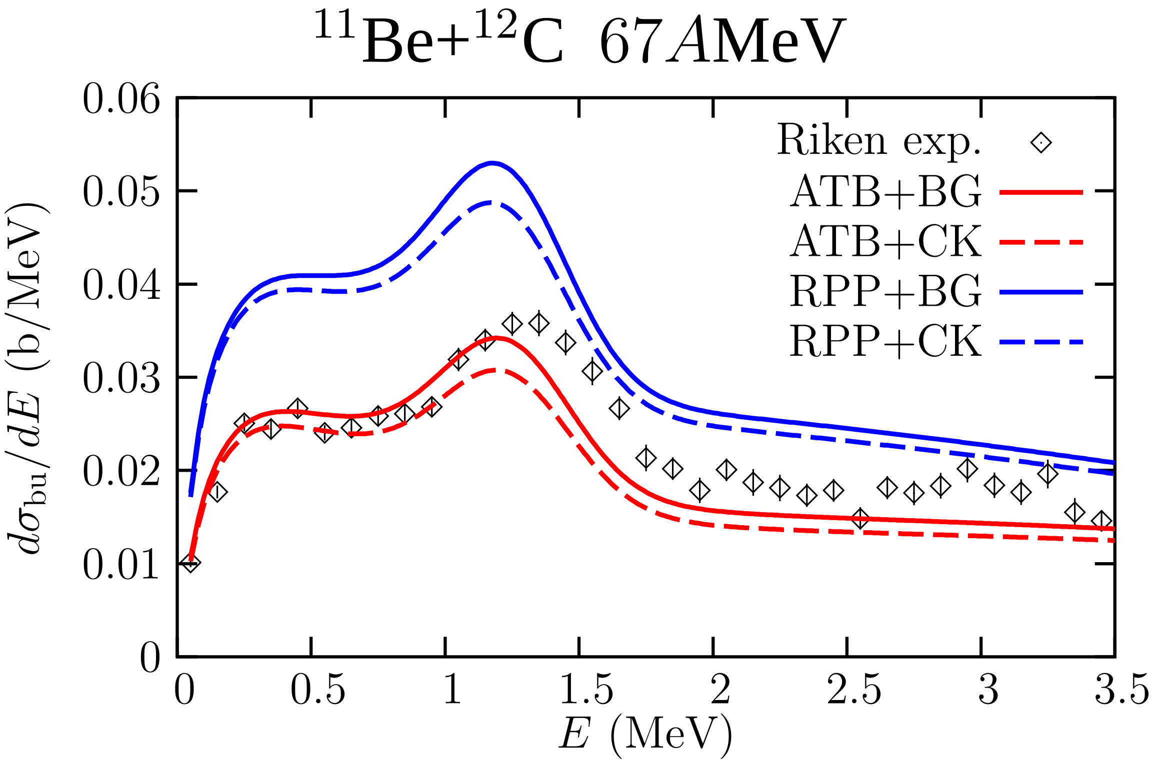
- Incoherent: $\widehat{\Psi}(\boldsymbol{r}, b, \phi_b, Z) = \widehat{\Psi}(\boldsymbol{r}, b, 0, Z)$
- Coherent: $\widehat{\Psi}(\boldsymbol{r}, b, \phi_b, Z) = e^{i(m_0 - l_z)\phi_b}\widehat{\Psi}(\boldsymbol{r}, b, 0, Z)$




Difference in pattern if $m \neq m_0$

Eikonal assumed incoherent

# Energy distribution




- Good agreement with experiment (No parameter)
- d5/2 resonance explained by simple 2-body model