Continuum Discretization and Coulomb Breakup. Application of the THO Formalism

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Outline of the presentation

- Introduction to the Transformed Harmonic Oscillator (THO) formalism
- Application of the THO model: analytical and numerical bound state cases.
- Coulomb Breakup: results for the reaction ^{8}B + ^{58}Ni.
- Orthogonal polynomials and continuum discretization.
- Concluding remarks.

Why Continuum Discretization?

- The consideration of the coupling to the continuum is important for the description of the structure and reactions of weakly-bound quantum systems.
- Difficult task due to the non-normalizable and continuum character of unbound eigenstates.
- A possible solution: discretize the continuum, handling “friendly” discrete and normalizable levels.
- Different approaches:
  1. Continuum Discretized Coupled Channels (CDCC) and Binning procedures.
  2. R-matrix calculations.
  3. Sturmian basis.
  4. Gaussian pseudostates.

THO as a Pseudostate Approach

- The pseudostate approach models the continuum of a two-body system by basis of discrete and finite set of square integrable functions.
  1. The system Hamiltonian is diagonalized in this basis.
  2. The resulting eigenstates model both the discrete and continuum spectra.
  3. The positive energy eigenstates are used as an input to CDCC calculations.
- The THO formalism generates a discrete representation of the continuum spectrum using the system ground state wave function as its unique input.

The THO Model

- The starting point is the ground state wave function of the two-body cluster $\phi_{0,l_0}(r) = rR_{0,l_0}(r)$.
- Computation of $s(r)$, a Local Scale Transformation (LST) that connects the system eigenfunction to a radial HO wave function.

$$
\begin{aligned}
s(r) \Rightarrow \phi_{0,l_0}(r) &= \sqrt{\frac{ds}{dr}}\phi^{HO}_{0,l_0} \\
\int_0^r dr' |\phi_{0,l_0}(r')|^2 &= \int_0^s ds' |\phi^{HO}_{0,l_0}(s')|^2 \\
\int_0^r dr' |\phi_{0,l_0}(r')|^2 &= \textit{erf}(s) - \frac{\exp\left(-s^2\right)}{\sqrt{\pi}} \sum_{i=0}^{l_0} 2^{l_0-i+1} \frac{s^{2l_0-2i+1}}{(2l_0-2i+1)!!}
\end{aligned}
$$

The THO Model

- Definition of a complete basis of orthonormal wave functions applying the LST to the HO wave functions

$$\phi_{n,l}(r) = [s(r)]^{l-l_0} L_n^{l+1/2}(s(r)^2)\phi_{0,l_0}(r) \quad n = 1, \ldots, \infty$$

- Diagonalization of the Hamiltonian in a N+1 dim truncated subset of the THO basis obtaining the functions: $\{\phi_{i,l}^{(N)}(r)\}_{i=0}^{N}$ such that $\hat{H}\phi_{i,l}^{(N)}(r) = e_{i,l}\phi_{i,l}^{(N)}(r)$.

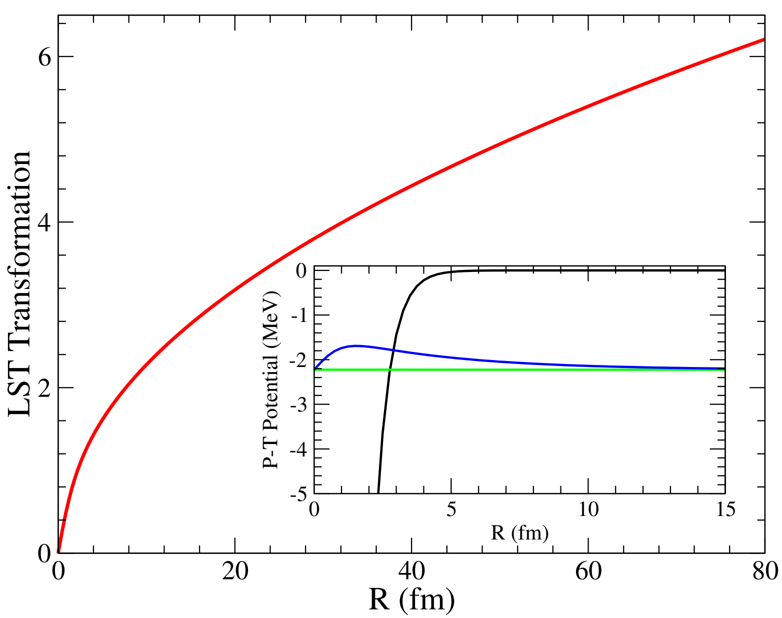
- By construction $\phi_{0,l_0}^{(N)}(r) = \phi_{0,l_0}(r)$ and $e_{0,l_0} = E_B$. The states with $e_{i,l} > 0$ are representative of the unbound part of the spectrum. Repeat for each value of angular momentum l involved in the system description.

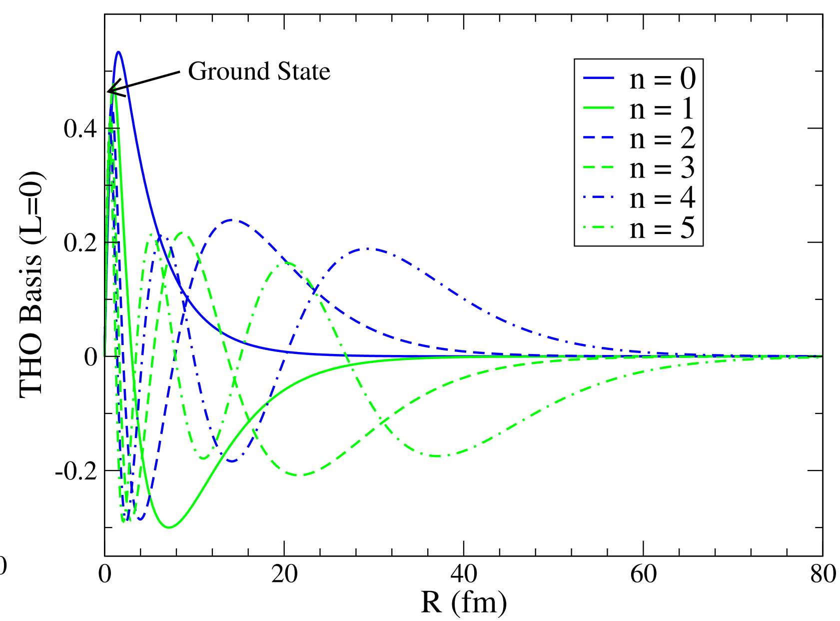
Application of the THO Model

- Analytical g.s. case: the deuteron proton-neutron interaction modeled with a 3D Pöschl-Teller potential.

$$V_{PT}(r) = -\frac{D}{\cosh^2(ar)} \qquad \phi_{0,0}(r) = \mathcal{N}_0 \frac{\sinh ar}{\cosh^j(ar)}$$

$$j = 1.2462 \quad a = \sqrt{2\mu B/\hbar^2(j-1)^2} = 0.9407 fm^{-1} \quad D = 102.7 MeV$$



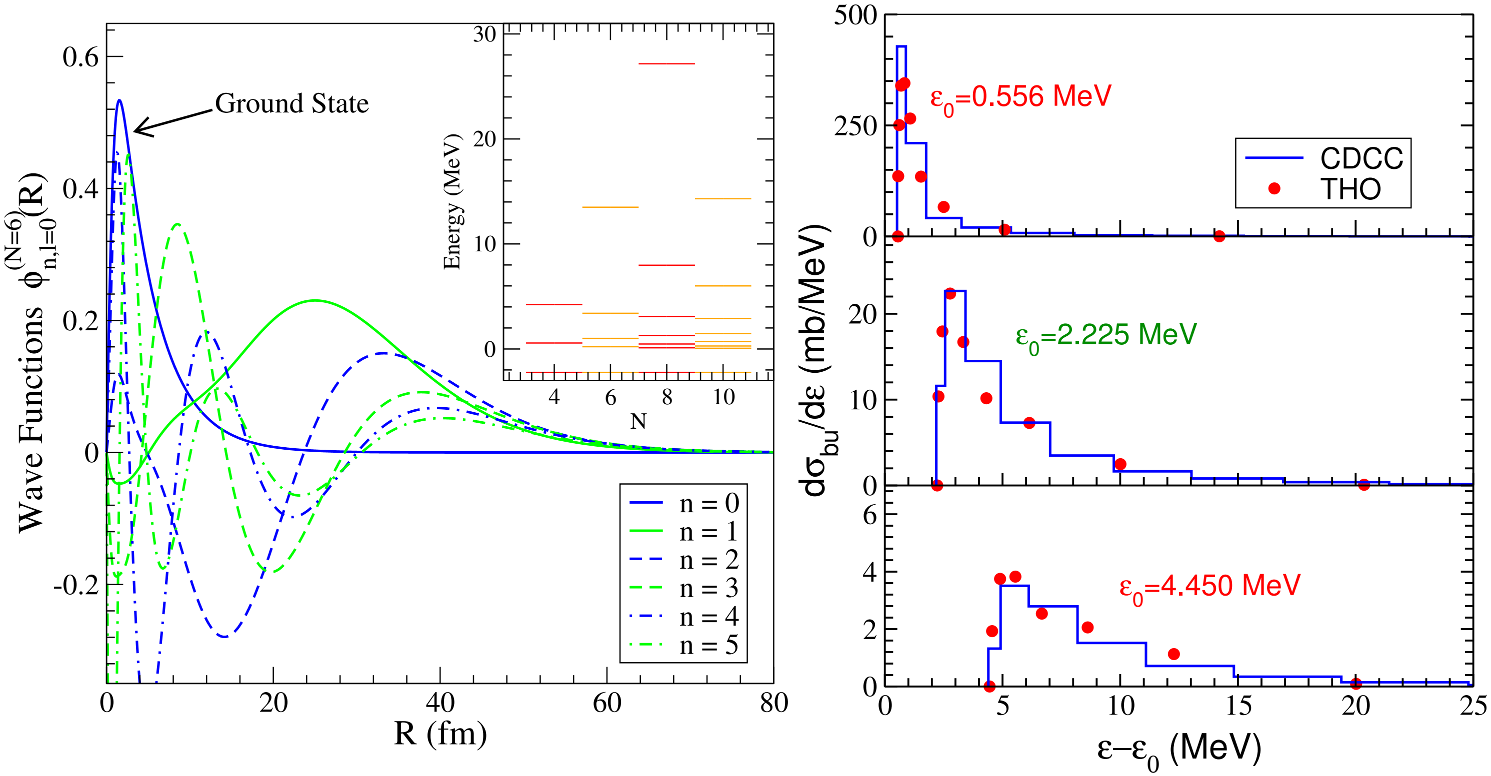


Application of the THO Model

- Deuteron s-wave breakup in d + ^{208}Pb at 50 MeV.

Left panel: THO eigenfunctions $\{\phi^{(N=5)}_{i,l=0}(r)\}^{5}_{i=0}$ and eigenvalues $\{e_{i,l=0}\}^{N=4,6,8,10}_{i=0}$.

Right panel: breakup cross section as a function of the deuteron excitation energy for three different binding energies.



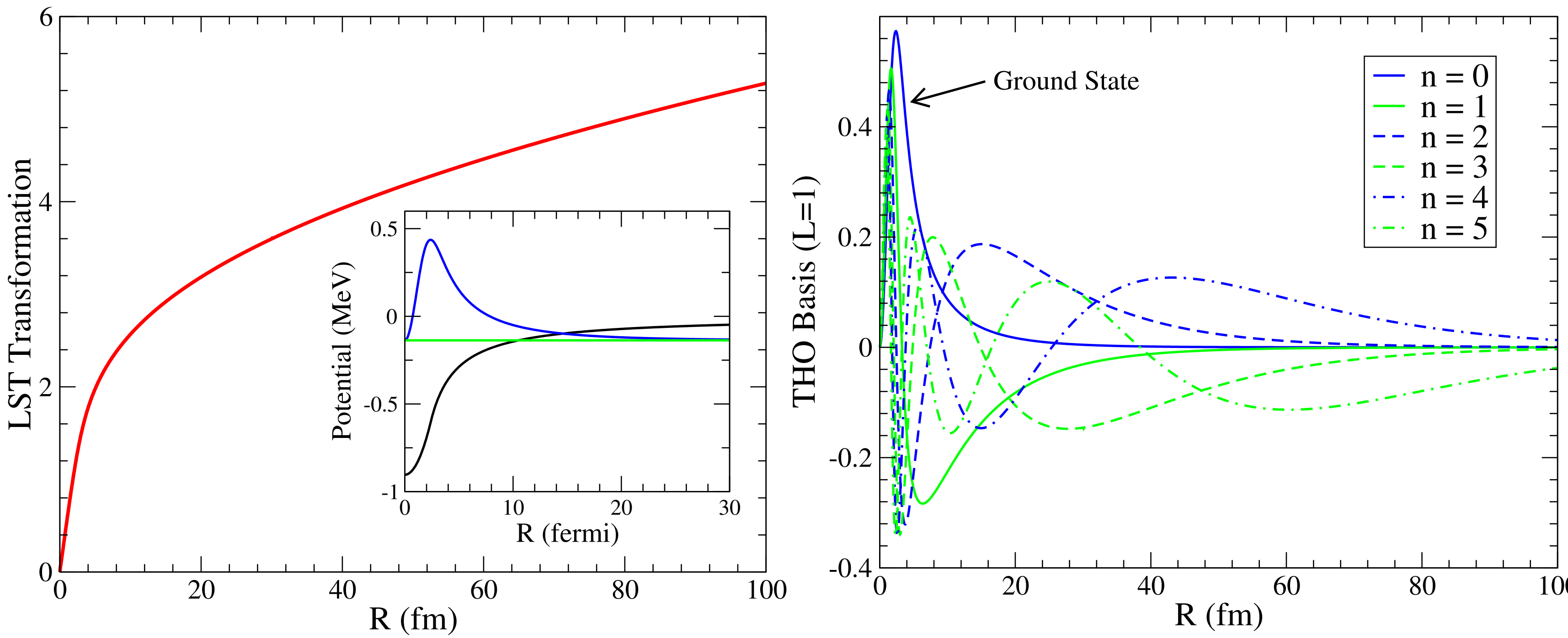
Application of the THO Model

- Numerical g.s. wave function case: the weakly bound system ^{8}B = p + ^{7}Be.

Woods Saxon potential[a] + Coulomb interaction.

Left panel: LST, potential and g.s. wavefunction.

Right panel: THO basis for $n = 0, \ldots, 5$ and $L = 1$.



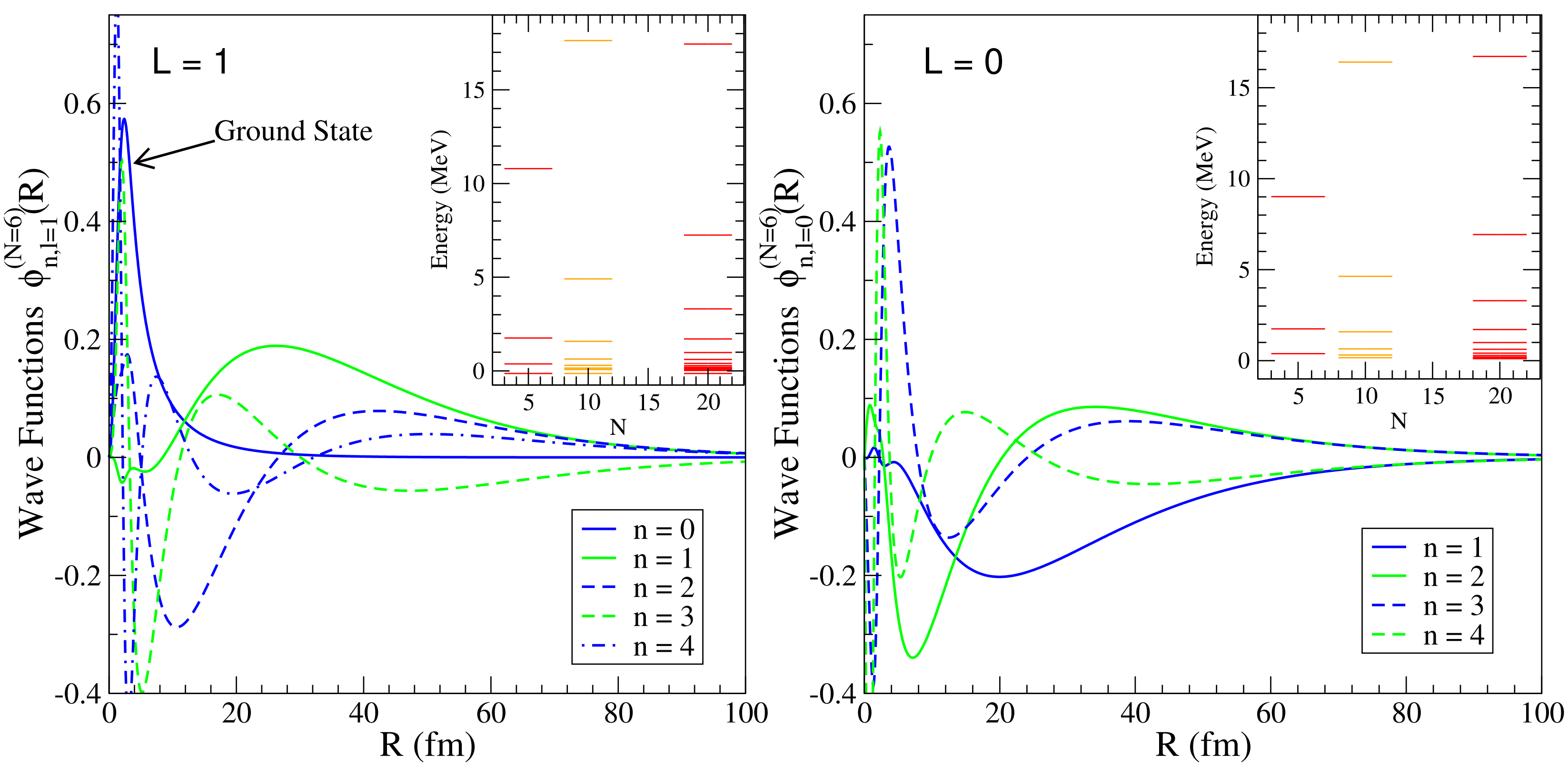
[a] H.Esbensen and G. Bertsch, Nucl.Phys. **A600** 47 (1999).

Application of the THO Model

- Numerical g.s. case: study of the reaction ^{8}B + ^{58}Ni at subcoulomb energies.

  Left panel: THO eigenfunctions $\{\phi_{i,l=1}^{(N=5)}(r)\}_{i=0}^{5}$ and eigenvalues $\{e_{i,l=1}\}_{i=0}^{N=5,10,20}$.

  Right panel: Id. for angular momentum $l = 0$.

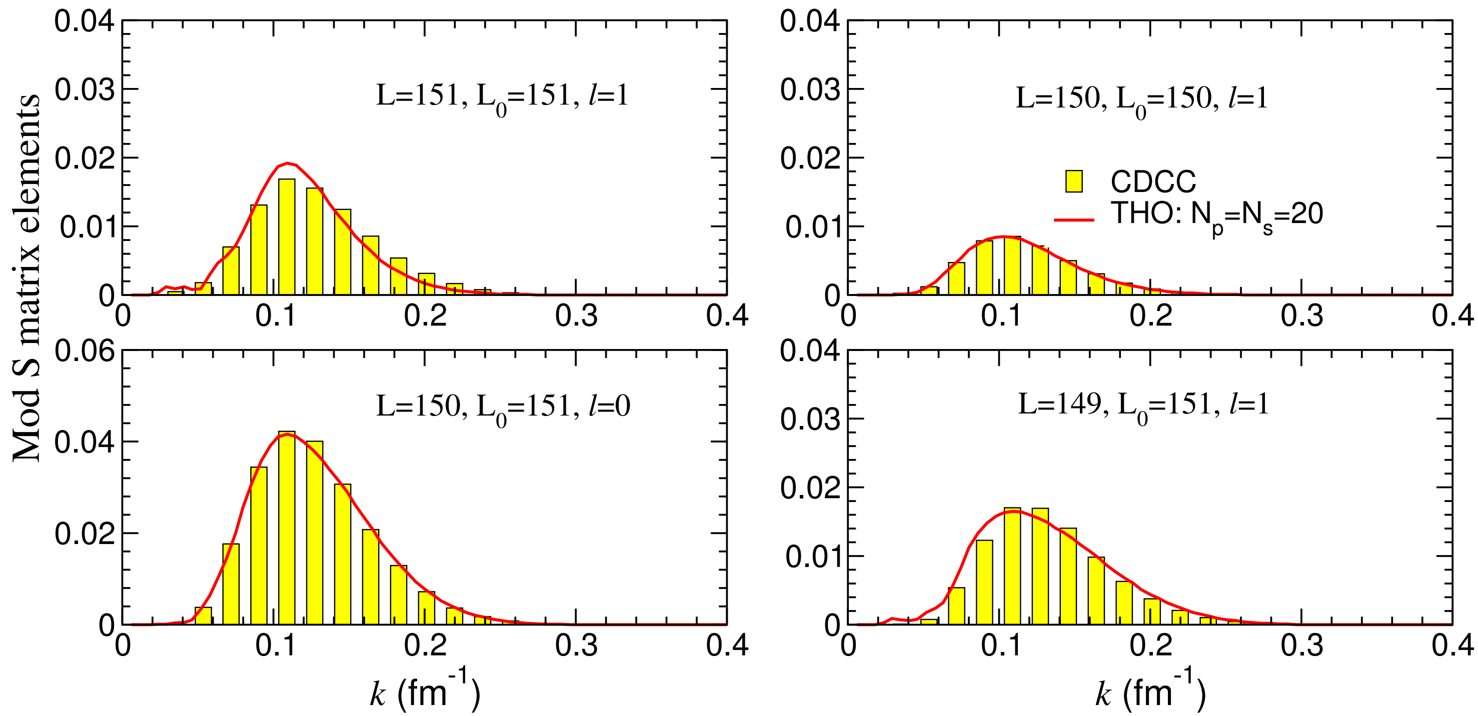


Coulomb Breakup: the reaction ^{8}B + ^{58}Ni

- Results obtained with the CDCC-THO approach.

Breakup S matrix elements for $J = 150$ as a function of the asymptotic p + ^{7}Be relative momentum. Superposition of discrete S-matrix elements[a]:

$$S_{\alpha':\alpha}(k) \simeq \sum_{i=1}^{N} \left\langle \phi_{k,l}^{(s)} \middle| \phi_{i,l}^{(N)} \right\rangle \hat{S}_{\alpha':\alpha}(k_i)$$



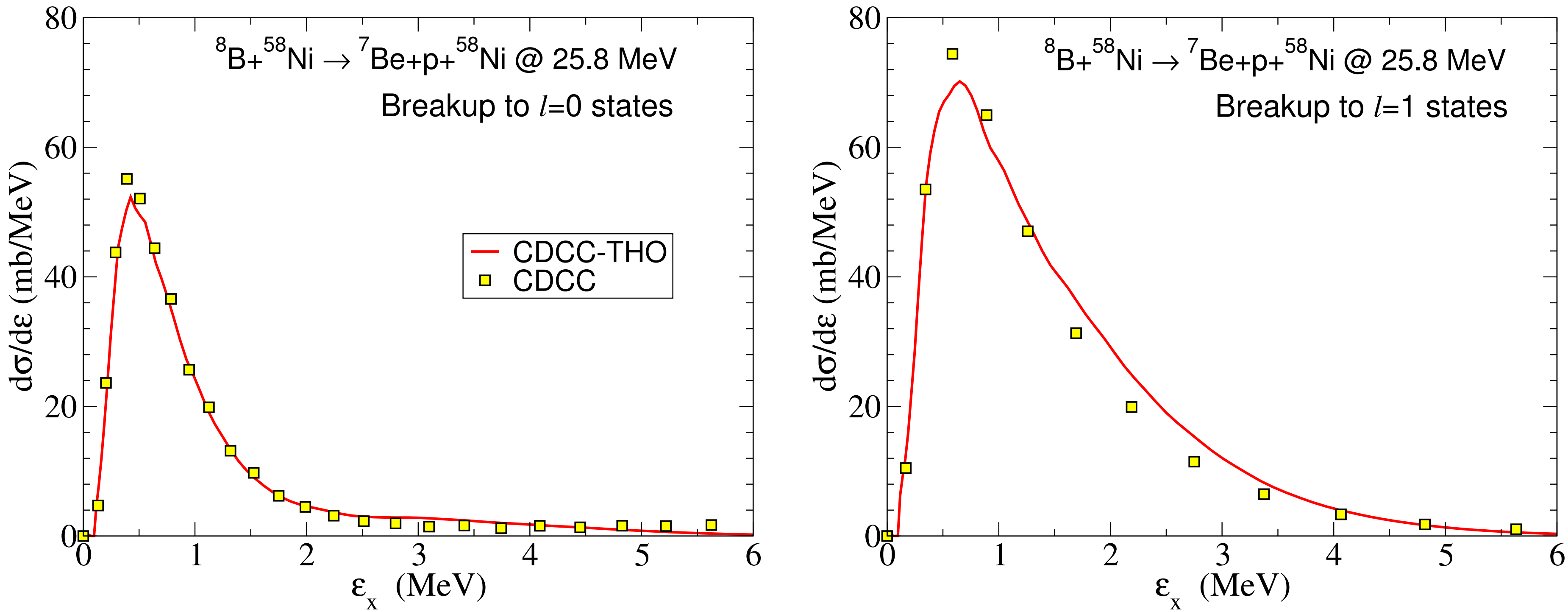
[a] T. Matsumoto et al. Phys. Rev. **C68** 064607 (2003).

Coulomb Breakup: the reaction ^{8}B + ^{58}Ni

- Results obtained with the CDCC-THO approach.

  Angle integrated breakup cross section dependence on the ^{8}B internal exc. energy.

  Basis dimension for CDCC-THO: 60 states ⇒ 30 states (0.01 MeV < E < 10 MeV).



Orthogonal Polynomials

- Proposed generalization of the THO formalism that is not dependent on the HO.
- The starting point is again the ground state wave function of the two-body cluster:

$$\phi_{0,l_0}(r) = rR_{0,l_0}(r)$$

- Definition of a weight function:

$$\omega_{l_0}(s) = \frac{dr}{ds}\left|\phi_{0,l_0}[r(s)]\right|^2$$

where $s(r)$ is an arbitrary continuous, single-valued, monotonically increasing or decreasing function in $r \in [a,b]$, the domain of $\phi_{0,l_0}(r)$.

Orthogonal Polynomials

- Define a family of orthogonal polynomials $P_{n,l}(s)$ such that
$$\int_a^b ds'\omega_l(s')P_{n,l}(s')P_{m,l}(s') = \frac{1}{\mathcal{N}_{n,l}\mathcal{N}_{m,l}}\delta_{n,m}$$
where for $l \neq l_0$, $\omega_l(s) = s^{2(l-l_0)}\omega_{l_0}(s)$.
- Construct an orthonormal basis based on the $P_{n,l}(s)$ polynomials with the correct behavior at the origin:
$$\phi^{OP}_{n,l}(r) = N_n\phi_{0,l_0}(r)r^{l-l_0}P_{n,l}[s(r)]$$
- Diagonalize the Hamiltonian for each partial wave and proceed in the same fashion than in the THO case.

Some References

- Some preliminary works using a LST:
  1. M.V. Stoitsov and I.Zh. Petkov, Ann.Phys.(N.Y.) **184** 121 (1988).
  2. M. Stoitsov, J. Dobaczewski, P. Ring, and S. Pittel, Phys. Rev. **C61**, 034311 (2000).
- The THO bibliography:
  1. F. Pérez-Bernal, I. Martel, J.M. Arias, and J. Gómez-Camacho, Phys. Rev. **A63**, 052111 (2001). (1 dimensional THO model)
  2. A.M. Moro et al., Phys. Rev. **C65**, 011602(R) (2001). (Deuteron s-wave breakup)
  3. F. Pérez-Bernal, I. Martel, J.M. Arias, and J. Gómez-Camacho, Phys. Rev. **A67**, 052108 (2003). (1D generalized model)
  4. M. Rodríguez-Gallardo, J.M. Arias, and J. Gómez-Camacho, Phys. Rev. **C69**, 034308 (2004). (Resonances)
  5. A.M. Moro, F. Pérez-Bernal, J.M. Arias, and J. Gómez-Camacho, nucl-th/0601097, accepted in Phys. Rev. **C**. (Coulom breakup)

Concluding Remarks

- The THO method can be implemented not only for analytical g.s. wave functions but also for numerically computed g.s. wave functions.
- The THO method can describe the continuum part of the spectrum for arbitrary partial waves.
- The applicability to situations dominated by long range interactions such as the dipole Coulomb has been proved.
- The results obtained with the CDCC-THO methodology agree in a satisfactory manner with those obtained using a standard CDCC method.
- The present method can describe resonances or the continuum of a 3-body system (M. Rodríguez-Gallardo).