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Evolution of proton states in neutron-rich Ca and Ar isotopes

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- **Motivation** **(exp. data and theoretical predictions)**
- **Adopted approach**
- **Inversion** **of proton states 2s1/2 and 1d3/2 in Ca isotopes? Analysis of the evolution of the states**
- **Ar isotopes. Inversion? Bubble structure?**
- **Conclusions**

Reduction of spin-orbit splitting for neutron p states in ^{47}Ar

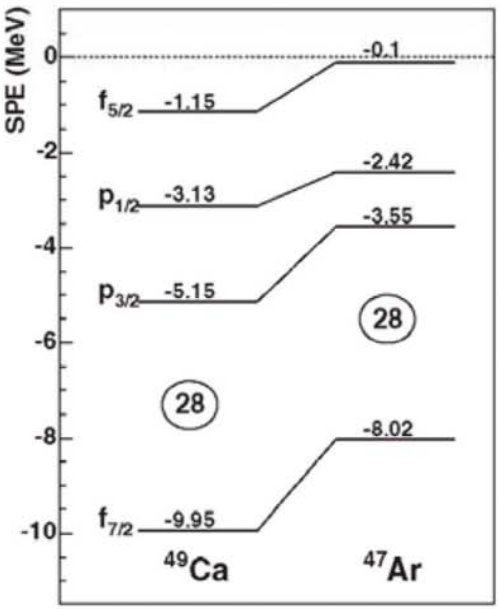


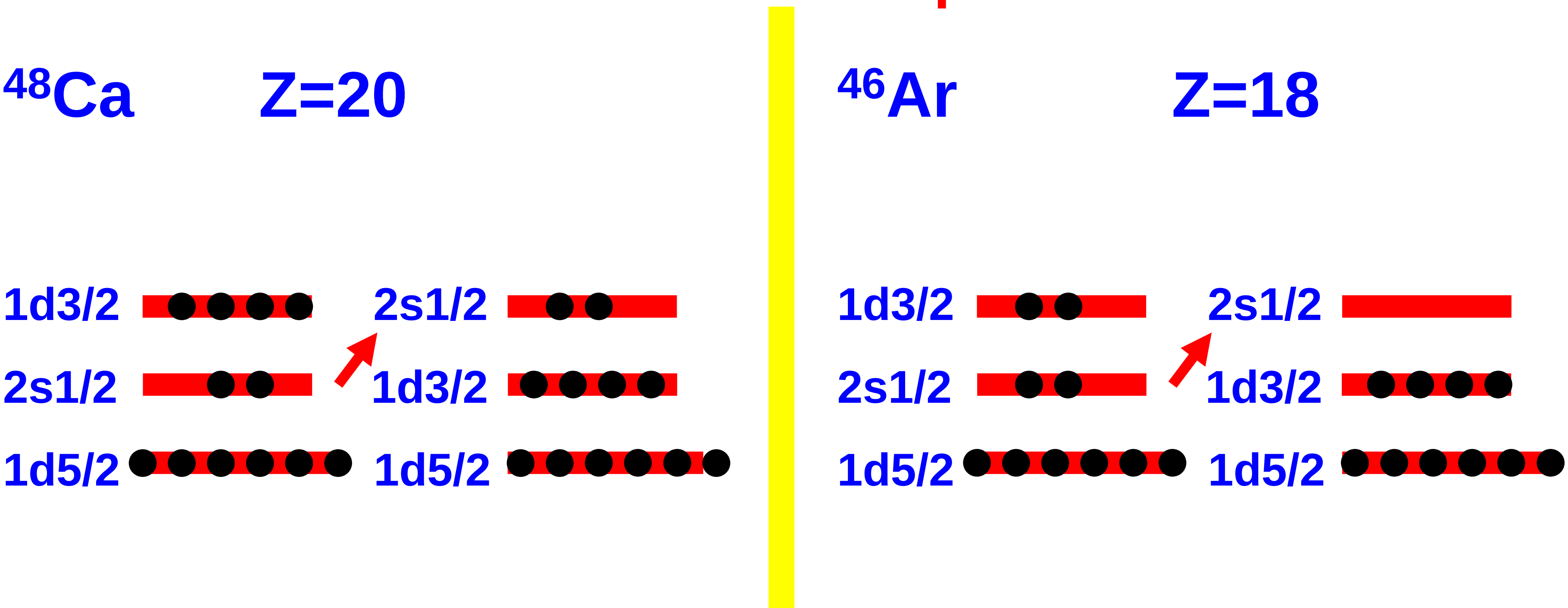
FIG. 3. Neutron single-particle energies (SPE) of the fp orbitals for the $^{47}Ar_{29}$ and $^{49}Ca_{29}$ nuclei (see text for details).

Transfer reaction $^{46}Ar(d,p)^{47}Ar$: energies and spectroscopic factors of neutron states p3/2, p1/2 and f5/2 in ^{47}Ar. Comparison with ^{49}Ca: reduction of ~ 8% and ~ 45% of the spin – orbit splitting for the f and p neutron states, respectively.

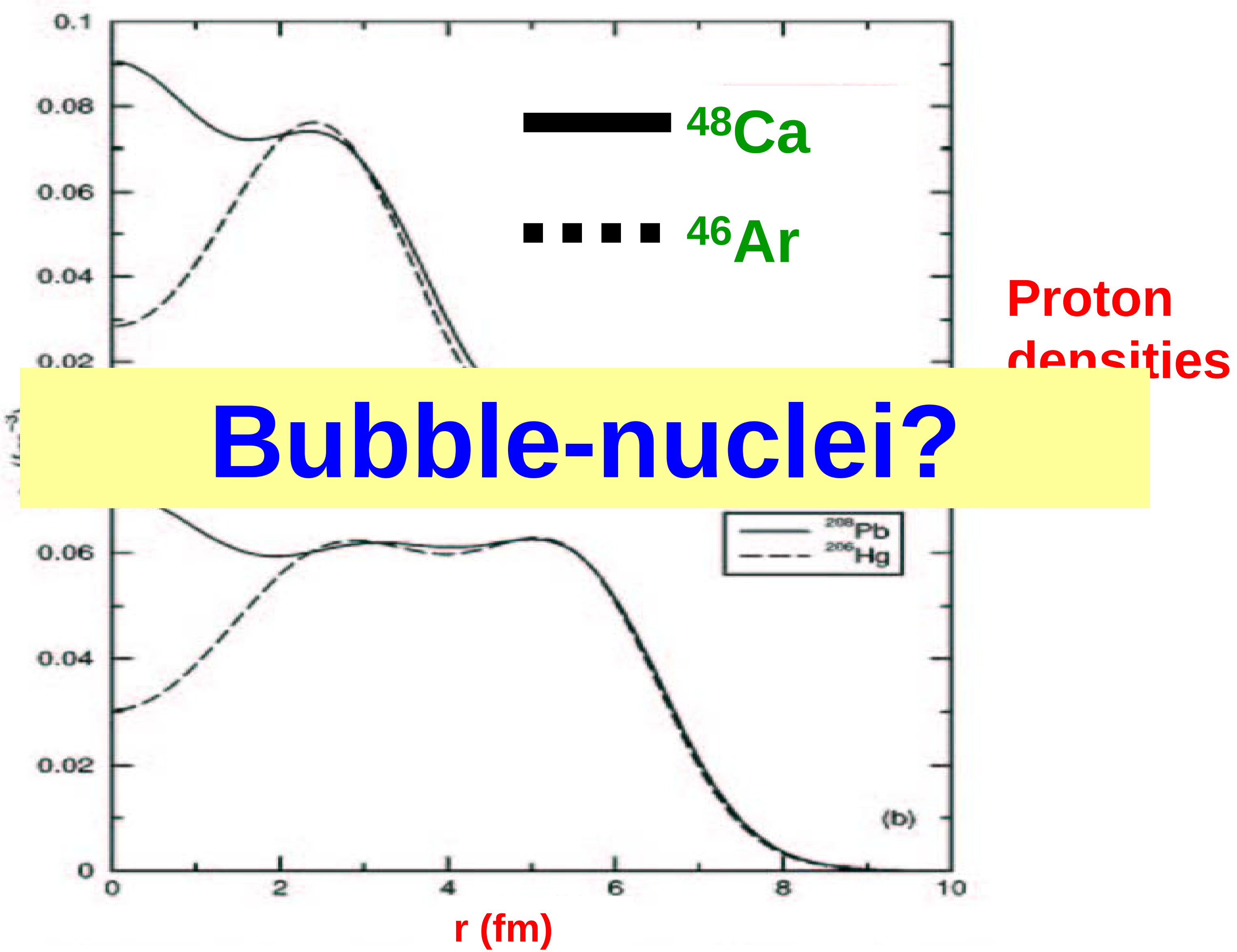
L. Gaudefroy, et al. PRL 97, 092501 (2006)

Theoretical analysis. Relativistic mean field (RMF). ^{48}Ca et ^{46}Ar

Inversion of s and d proton states



B.G. Todd-Rutel, et al., PRC 69, 021301 (R) (2004)



B.G. Todd-Rutel, et al., PRC 69, 021301 (R) (2004)

B.G. Todd and J. Piekarewicz, PRC 67, 044317 (2003)

Two questions

- **Predictions obtained in the framework of the non relativistic mean field (not only in ^{48}Ca and ^{46}Ar)?**

- **Bubble structure in Ar isotopes?**

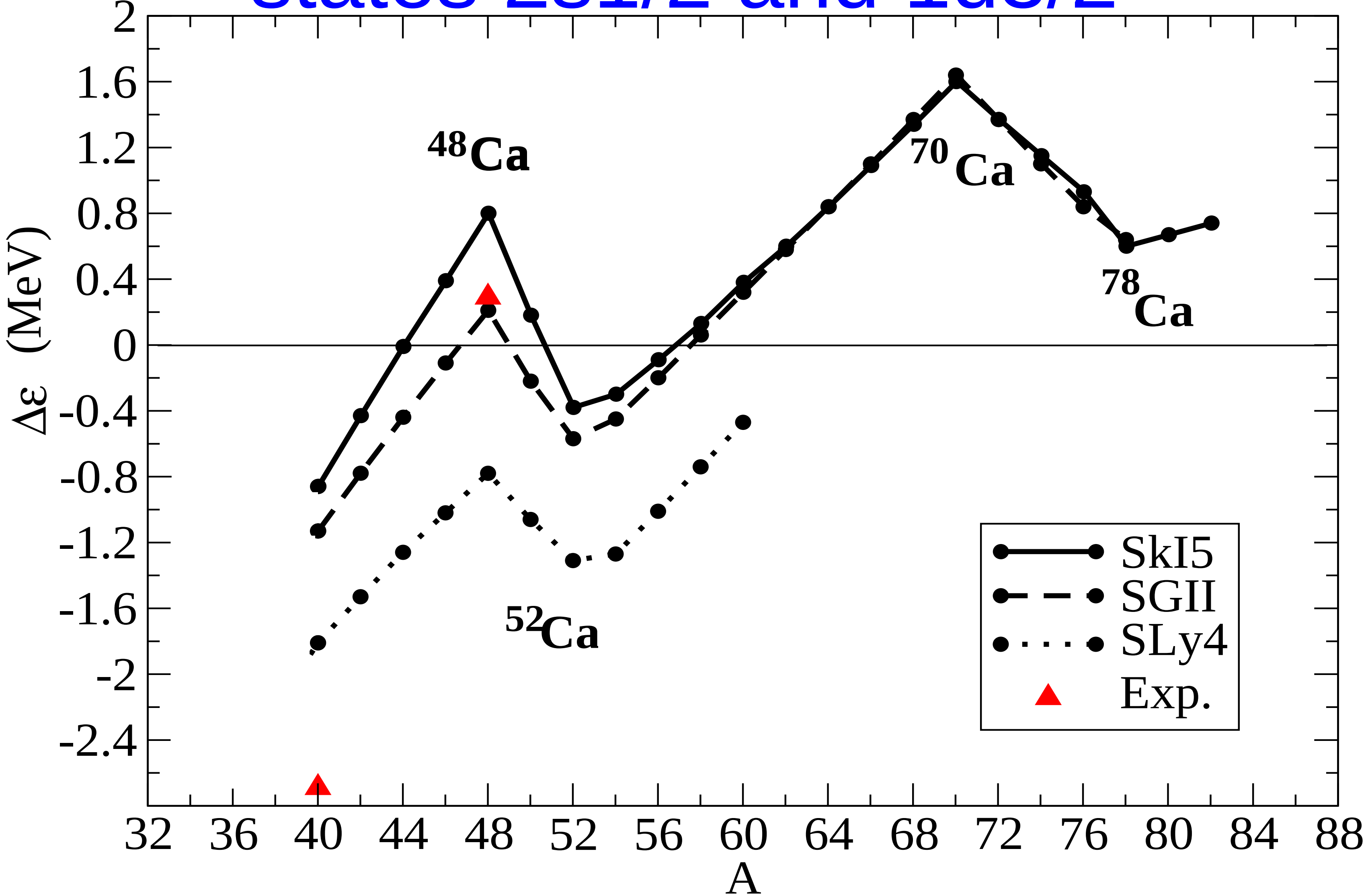
Adopted approach:

Non relativistic mean field

Hartree-Fock with Skyrme

Z . Ma -> Relativistic Mean Field (RMF)

Energy difference between the states 2s1/2 and 1d3/2



Grasso, Ma, Khan, Margueron, Van Giai, PRC 76, 044319 (2007)

Spin – orbit potential

Non relativistic case and standard Skyrme forces

$$V_{SO}^{q} = \frac{W_0}{2}\left(\nabla\rho + \nabla\rho_q\right) = \frac{W_0}{2}\left(2\nabla\rho_q + \nabla\rho_{q'}\right)$$

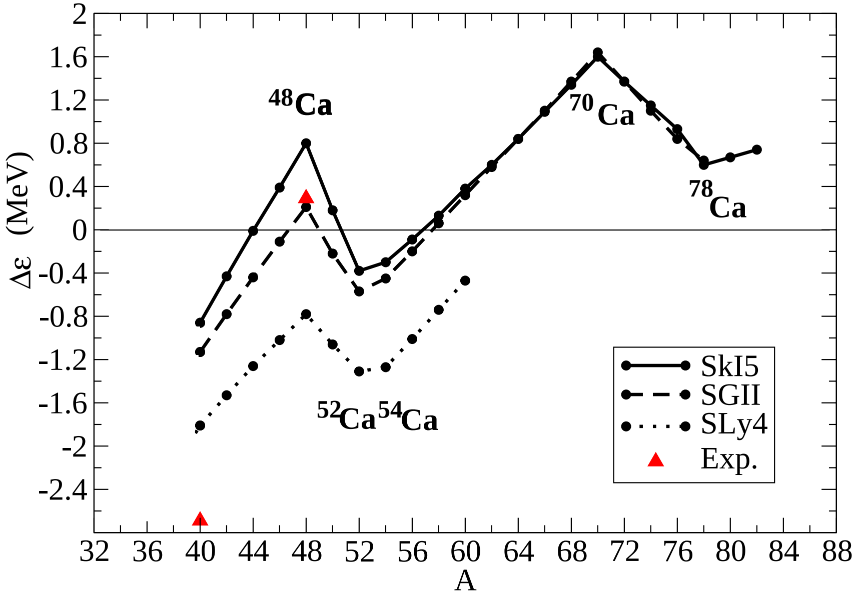
q, q’ -> proton or neutron

Relativistic case

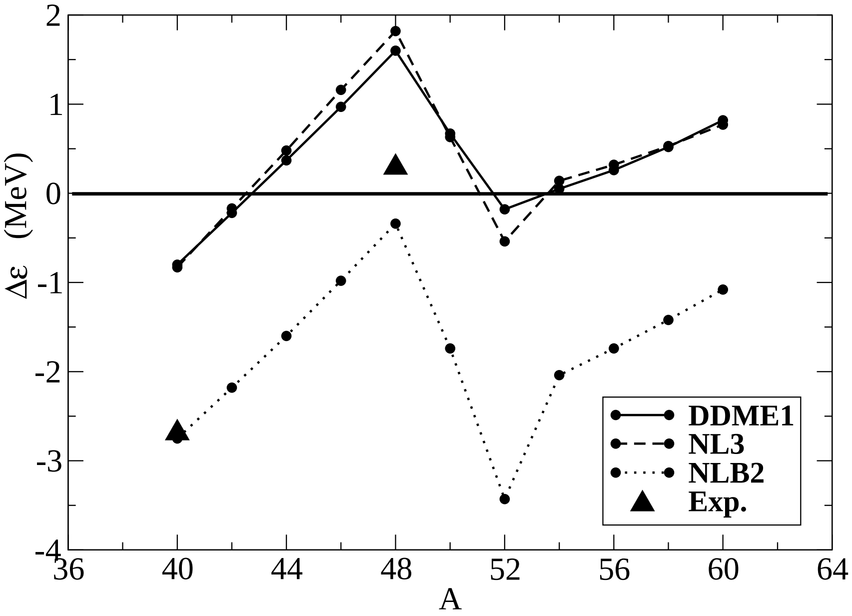
The potential is proportional to $\nabla\rho = \nabla\rho_q + \nabla\rho_{q'}$

Energy difference between the states 2s1/2 and 1d3/2

Non relativistic mean field



Relativistic mean field



Grasso, Ma, **Khan, Margueron, Van Giai, PRC 76, 044319 (2007)**

Hartree-Fock equations with the equivalent potential.

$$\frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2}\psi(r)+\frac{l(l+1)}{r^2}\psi(r)\right]+V_{eq}^{lj}(r,\varepsilon)\psi(r)=\varepsilon\psi(r)$$

Equivalent potential:

$$V_{eq}^{lj}(r,\varepsilon)=\frac{m^*(r)}{m}U_0(r)-\frac{m^{*2}(r)}{2m\hbar^2}\left(\frac{\hbar^2}{2m^*(r)}\right)^{'2}+\frac{m^*(r)}{2m}\left(\frac{\hbar^2}{2m^*(r)}\right)^{''}+\left[1-\frac{m^*(r)}{m}\right]\varepsilon+\frac{m^*(r)}{m}U_{so}^{lj}(r)$$

Central term

$$V_{eq}^{lj}(r,\varepsilon) = \frac{m^*(r)}{m} U_0(r) - \frac{m^{*2}(r)}{2m\hbar^2}\left(\frac{\hbar^2}{2m^*(r)}\right)'^2 + \frac{m^*(r)}{2m}\left(\frac{\hbar^2}{2m^*(r)}\right)'' + \left[1 - \frac{m^*(r)}{m}\right]\varepsilon + \frac{m^*(r)}{m} U_{so}^{lj}(r)$$

V_{eq}^{centr}

$$\varepsilon = \langle T \rangle + \langle V_{eq}^{centr} \rangle + \langle V_{so} \rangle + \left\langle 1 - \frac{m^*(r)}{m} \right\rangle \varepsilon$$

$$
\varepsilon_s - \varepsilon_d = \left[\frac{1}{\langle m^*(r)/m \rangle_s} \langle T \rangle_s - \frac{1}{\langle m^*(r)/m \rangle_d} \langle T \rangle_d \right]
$$

Kinetic contribution

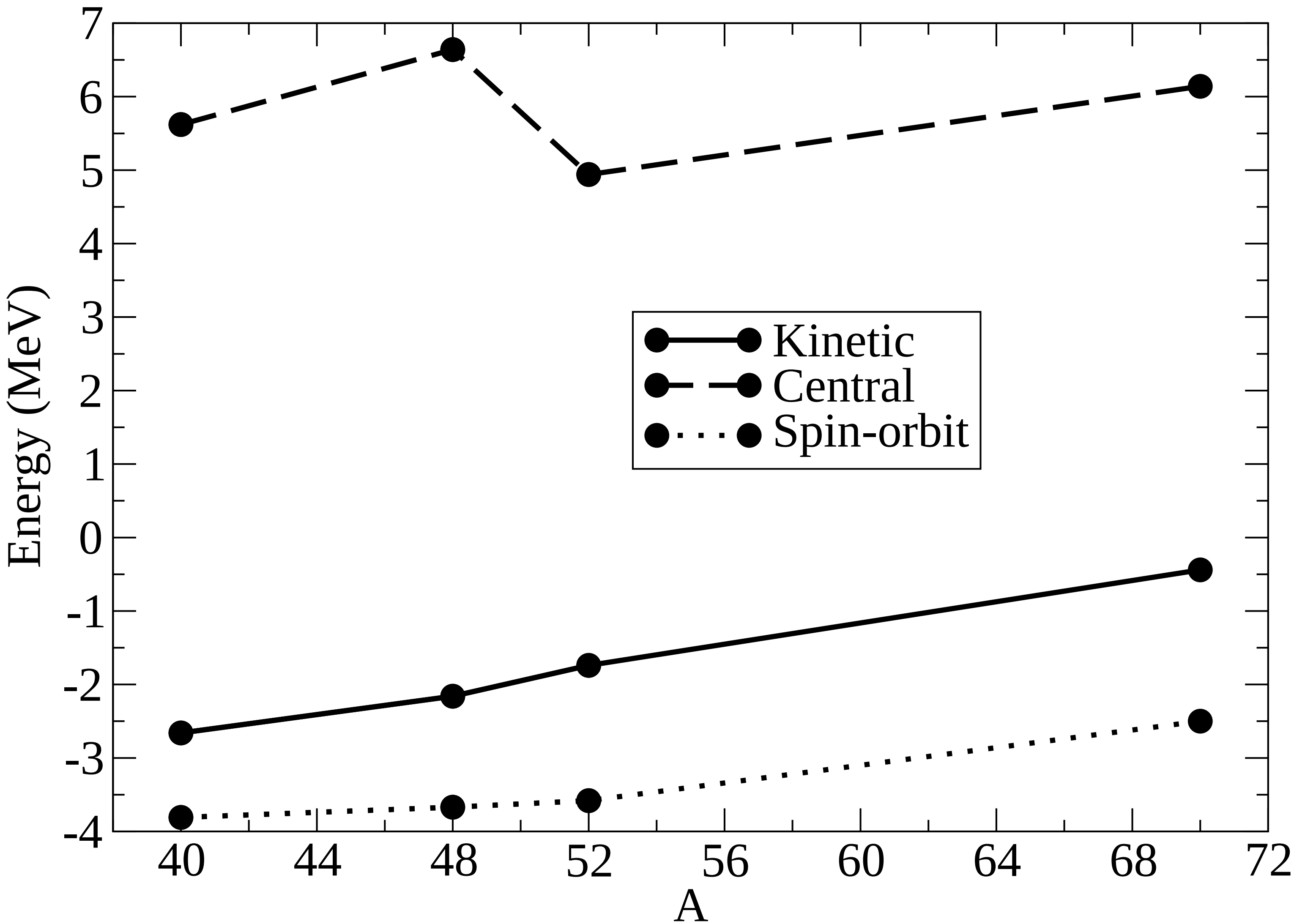
$$
+ \left[\frac{1}{\langle m^*(r)/m \rangle_s} \langle V_{eq}^{centr} \rangle_s - \frac{1}{\langle m^*(r)/m \rangle_d} \langle V_{eq}^{centr} \rangle_d \right]
$$

Central contribution

$$
- \frac{1}{\langle m^*(r)/m \rangle_d} \langle V_{so} \rangle_d
$$

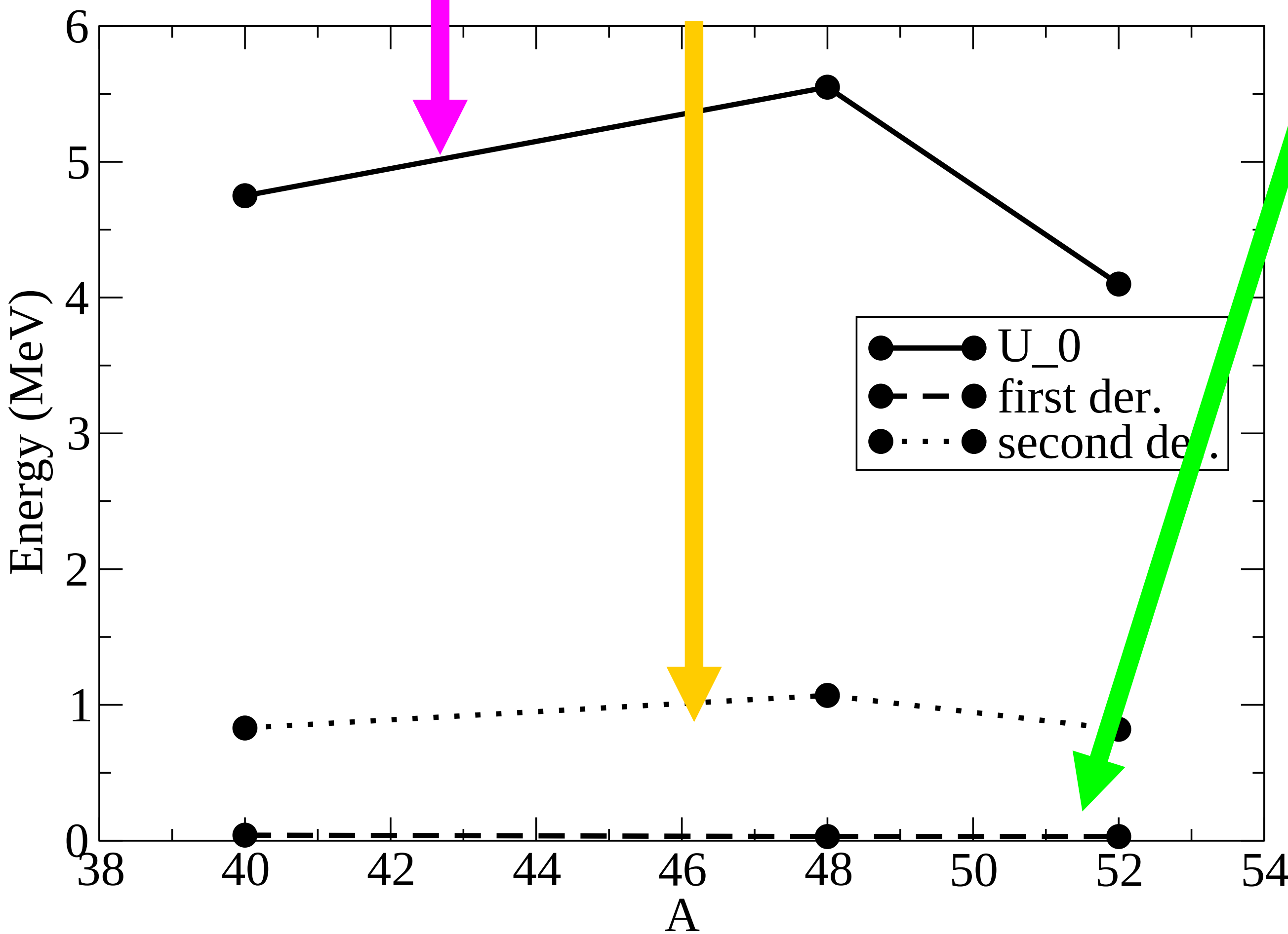
Spin-orbit contribution

Kinetic, central and spin – orbit contributions to the energy difference between the states 2s1/2 and 1d3/2



Grasso, Ma, Khan, Margueron, Van Giai, PRC 76, 044319 (2007)

$$V_{eq}^{centr}(r,\varepsilon) = \frac{m^*(r)}{m} U_0(r) - \frac{m^{*2}(r)}{2m\hbar^2}\left(\frac{\hbar^2}{2m^*(r)}\right)'^2 + \frac{m^*(r)}{2m}\left(\frac{\hbar^2}{2m^*(r)}\right)''$$



Grasso, Ma, Khan, Margueron, Van Giai, PRC 76, 044319 (2007)

Important contributions of the HF potential

Central term

$$\frac{1}{2}t_0\left[(2+x_0)\rho-(1+2x_0)\rho_q\right]$$

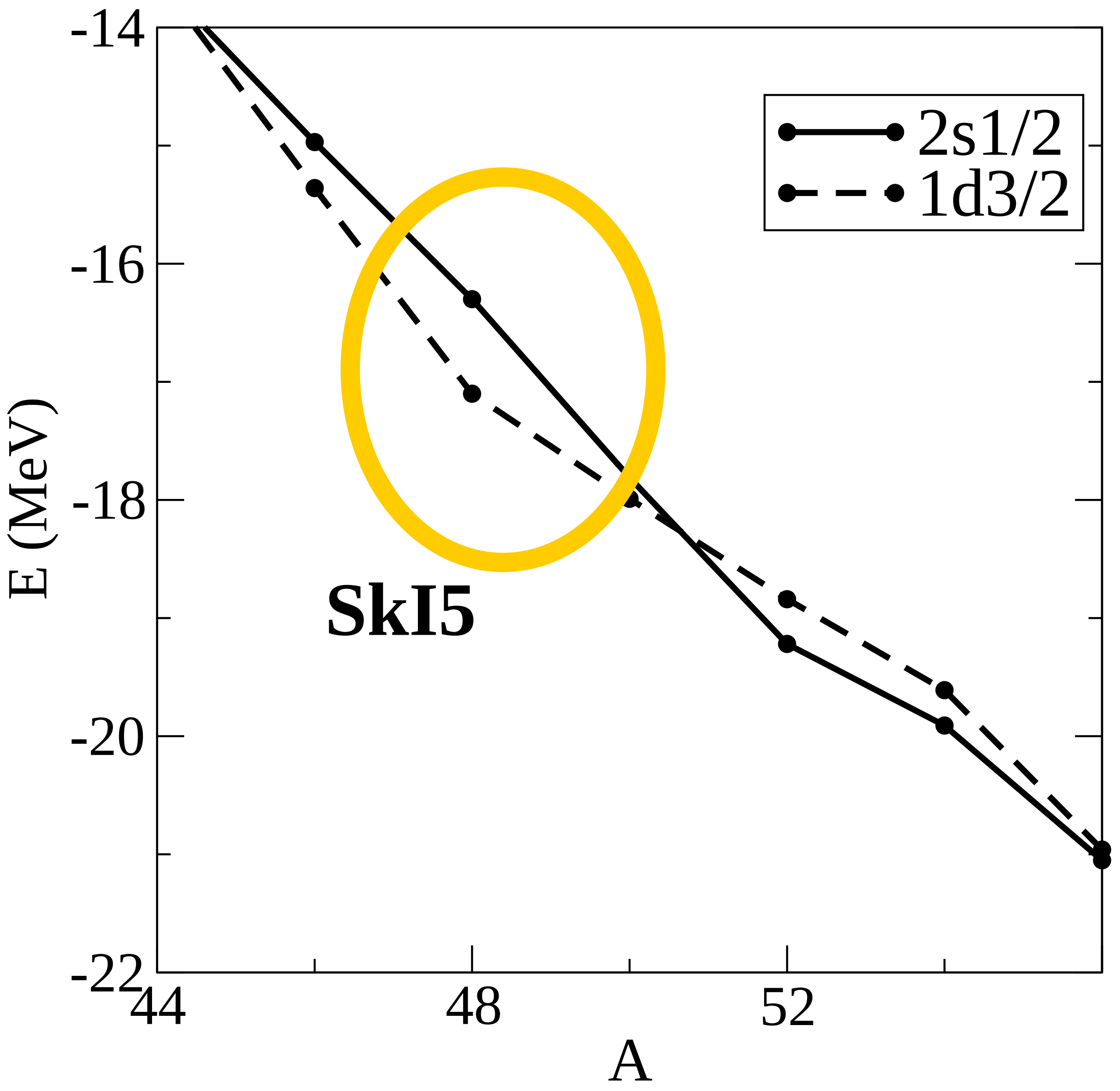
It favors the inversion

Density-dependent term

$$\frac{1}{24}t_3\left\{(2+x_3)(2+\alpha)\rho^{\alpha+1}-(2x_3+1)\left[2\rho^{\alpha}\rho_q+\alpha\rho^{\alpha-1}\left(\rho_p^2+\rho_n^2\right)\right]\right\}$$

Against the inversion

2s1/2 and 1d3/2 states near ^{48}Ca



…and the tensor contribution?

- **Shell model : T. Otsuka, et al., PRL 95, 232502 (2005)**

- **Relativistic mean field: RHFB : W. Long, et al., PLB 640, 150 (2006)**

- **Non relativistic mean field:**
- **Skyrme : G. Colò, et al., PLB 646, 227 (2007)**
- **Gogny : T. Otsuka, et al., PRL 97, 162501 (2006)**

Variation of the energy density (dependence on *J*)

$$\Delta H = \frac{1}{2}\alpha\left(J_n^2 + J_p^2\right) + \beta J_n J_p$$

J -> spin density

$$J_q(r) = \frac{1}{4\pi r^3}\sum_i \left(2j_i + 1\right)\left[j_i\left(j_i + 1\right) - l_i\left(l_i + 1\right) - \frac{3}{4}\right]v_i^2(r)$$

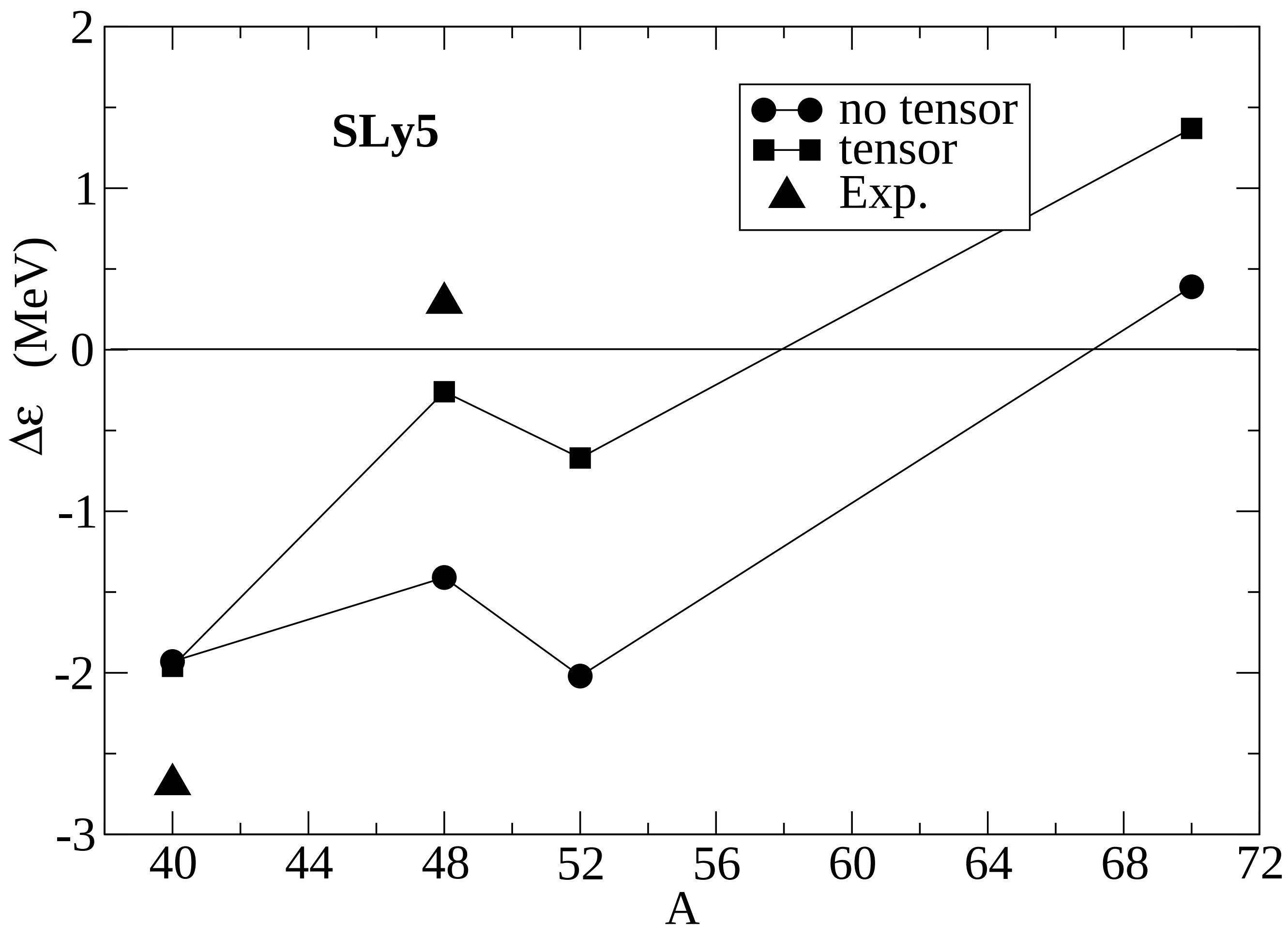
The spin – orbit potential is modified:

$$U_{SO}^q = \frac{W_0}{2}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha J_q + \beta J_{q'}\right)$$

$$\alpha = \alpha_C + \alpha_T \qquad \beta = \beta_C + \beta_T$$

$$\alpha_C = \frac{1}{8}\left(t_1 - t_2\right) - \frac{1}{8}\left(t_1 x_1 + t_2 x_2\right) \qquad \beta_C = -\frac{1}{8}\left(t_1 x_1 + t_2 x_2\right)$$

Effect due to the tensor contribution with SLy5



Grasso, Ma, Khan, Margueron, Van Giai, PRC 76, 044319 (2007)

HF- Skyrme for Ar isotopes (Z=18)

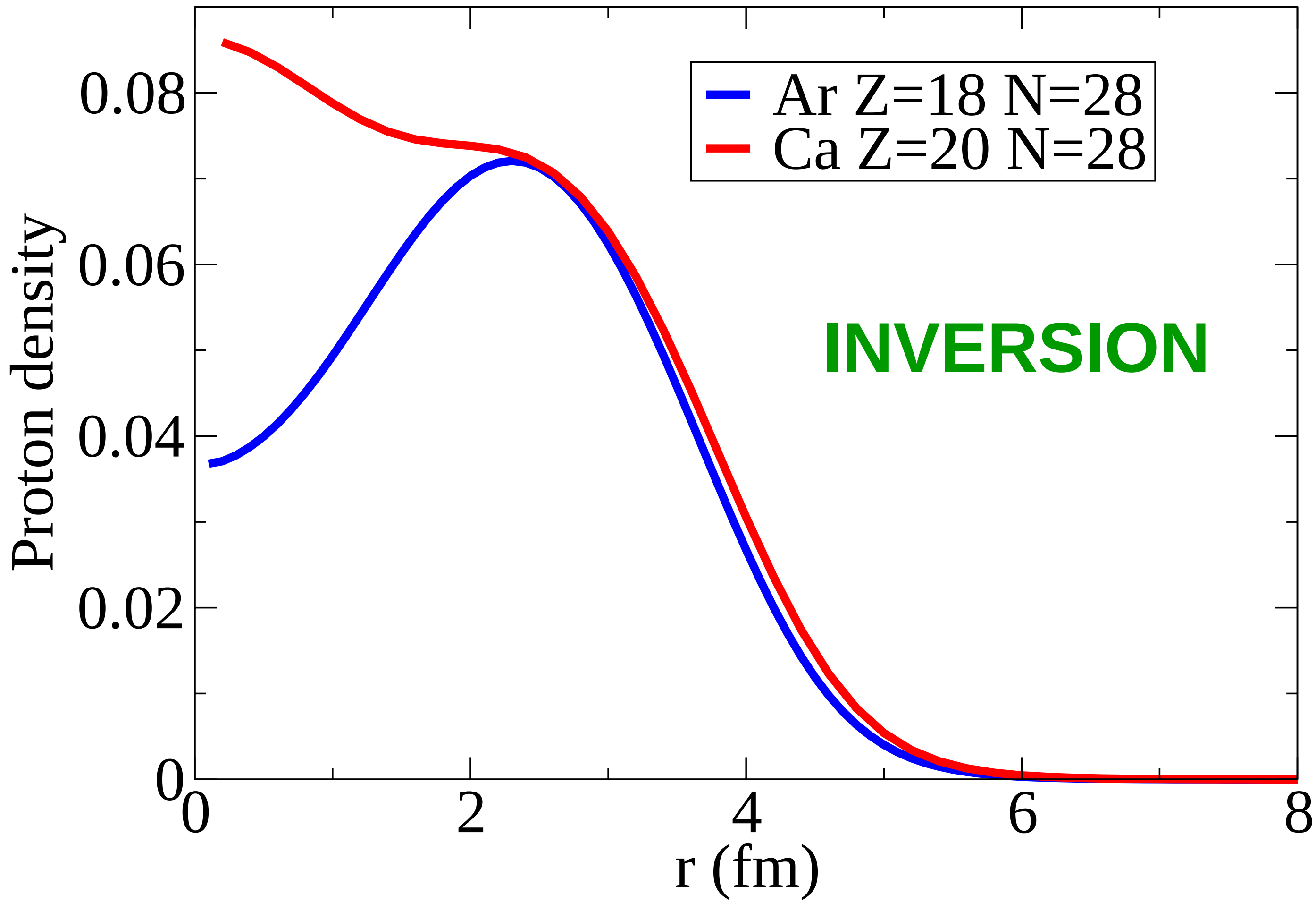
Inversion with SkI5 in ^{46}Ar and for very neutron-rich isotopes starting from ^{60}Ar up to the drip line (^{68}Ar)

SLy4 : no inversion

Bubbles?

- **Wilson in 1946** (H.A. Wilson, Phys. Rev. 69 (1946) 538)
- C.Y. Wong, Phys. Lett. 41B (1972); Ann. Phys. (N.Y.) 77 (1973) 279
- First microscopic calculation **HF+BCS**: X. Campi and D.W.L. Sprung, Phys. Lett. 46B (1973), 291 (^{36}Ar, ^{200}Hg)
- **HFB with Gogny** D1S, J. Dechargé, et al. Nucl. Phys. A716 (2003) 55 (in **superheavy** nuclei)

HF proton density in ^{46}Ar with SkI5



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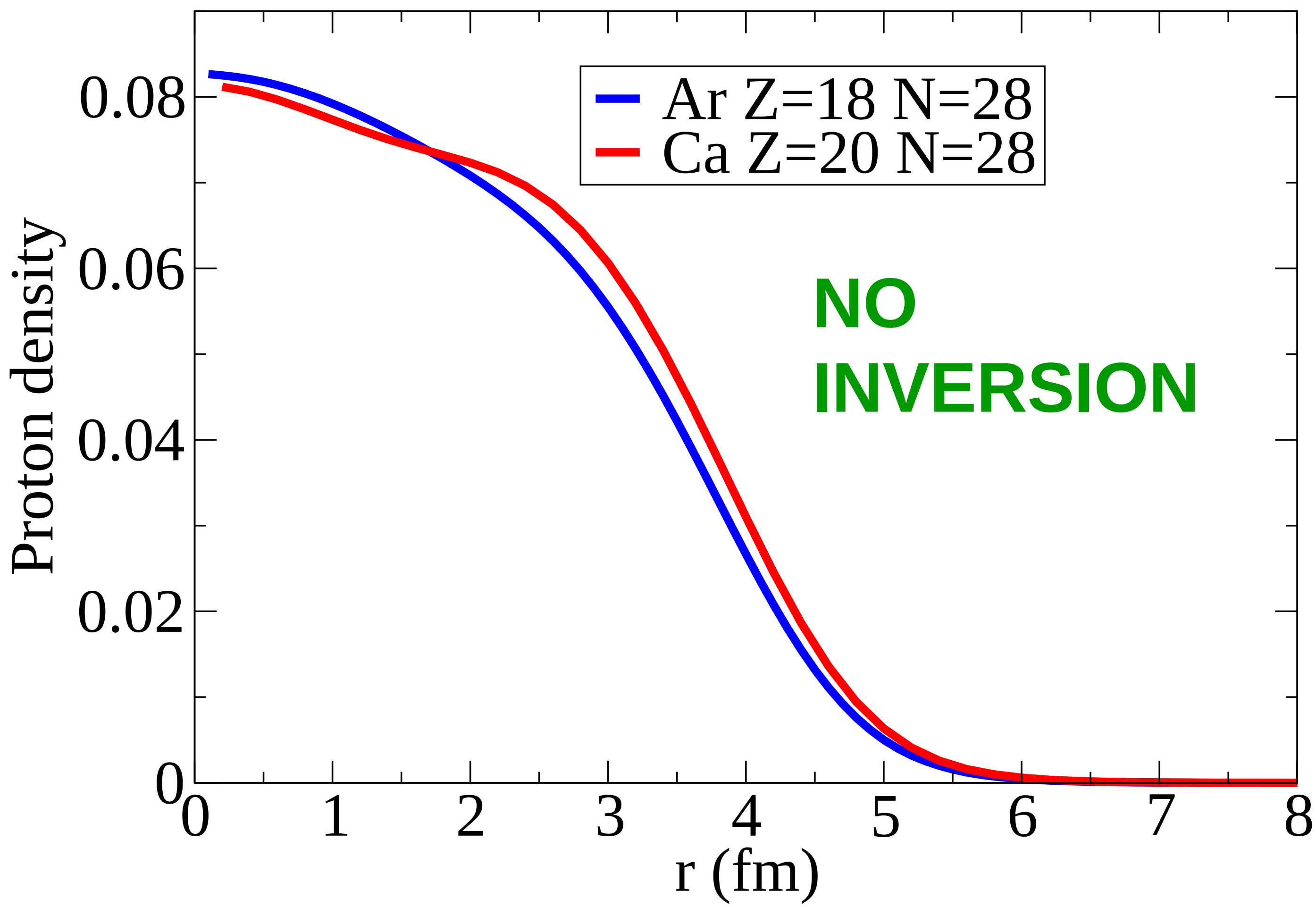
HF proton density in ^{46}Ar with SLv4



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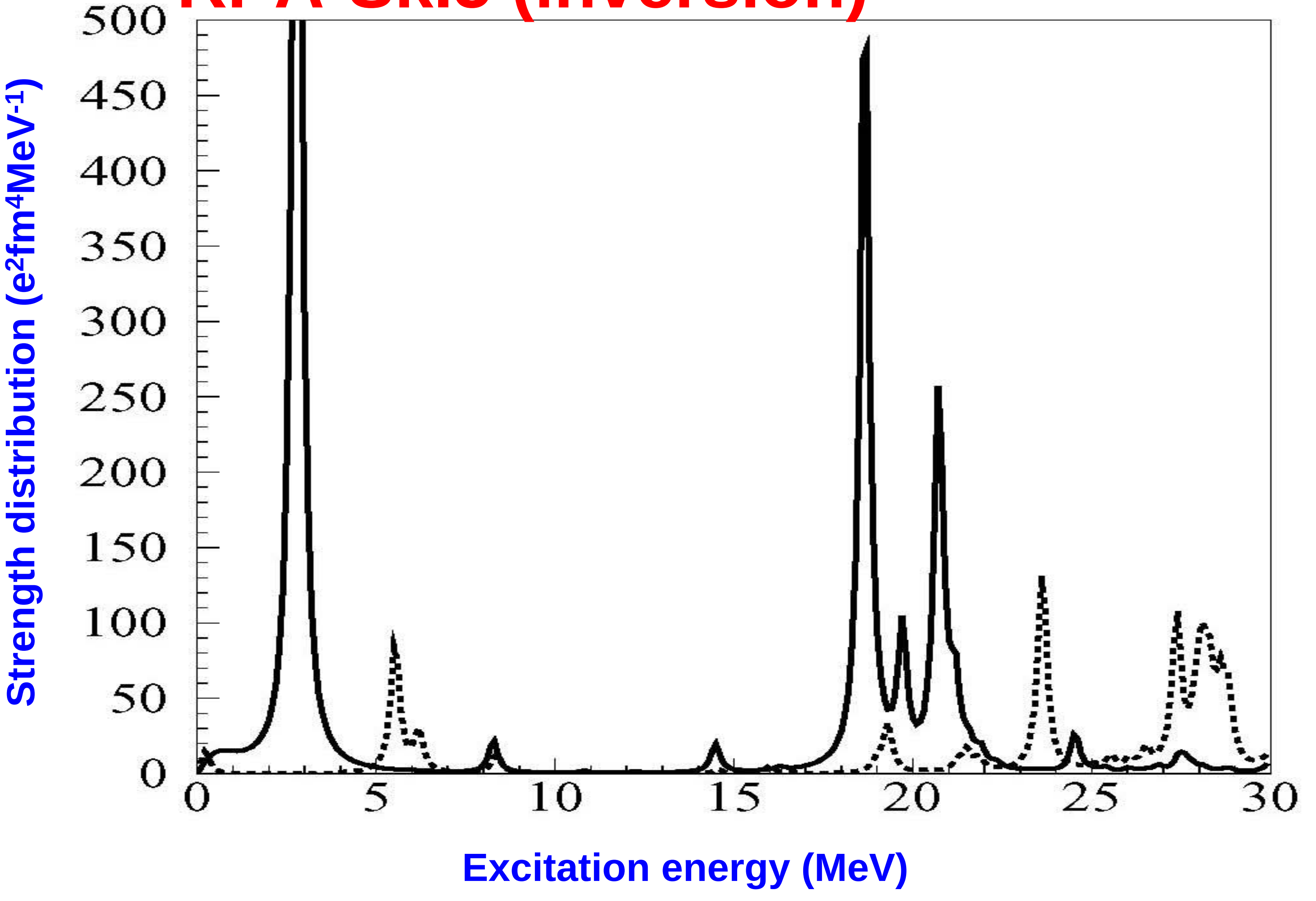
Proprieties associated to a bubble structure. Which experimental signals?

One possible direction: the excited states.

RPA

Quadrupole excitations in ^{46}Ar

RPA-SkI5 (inversion)



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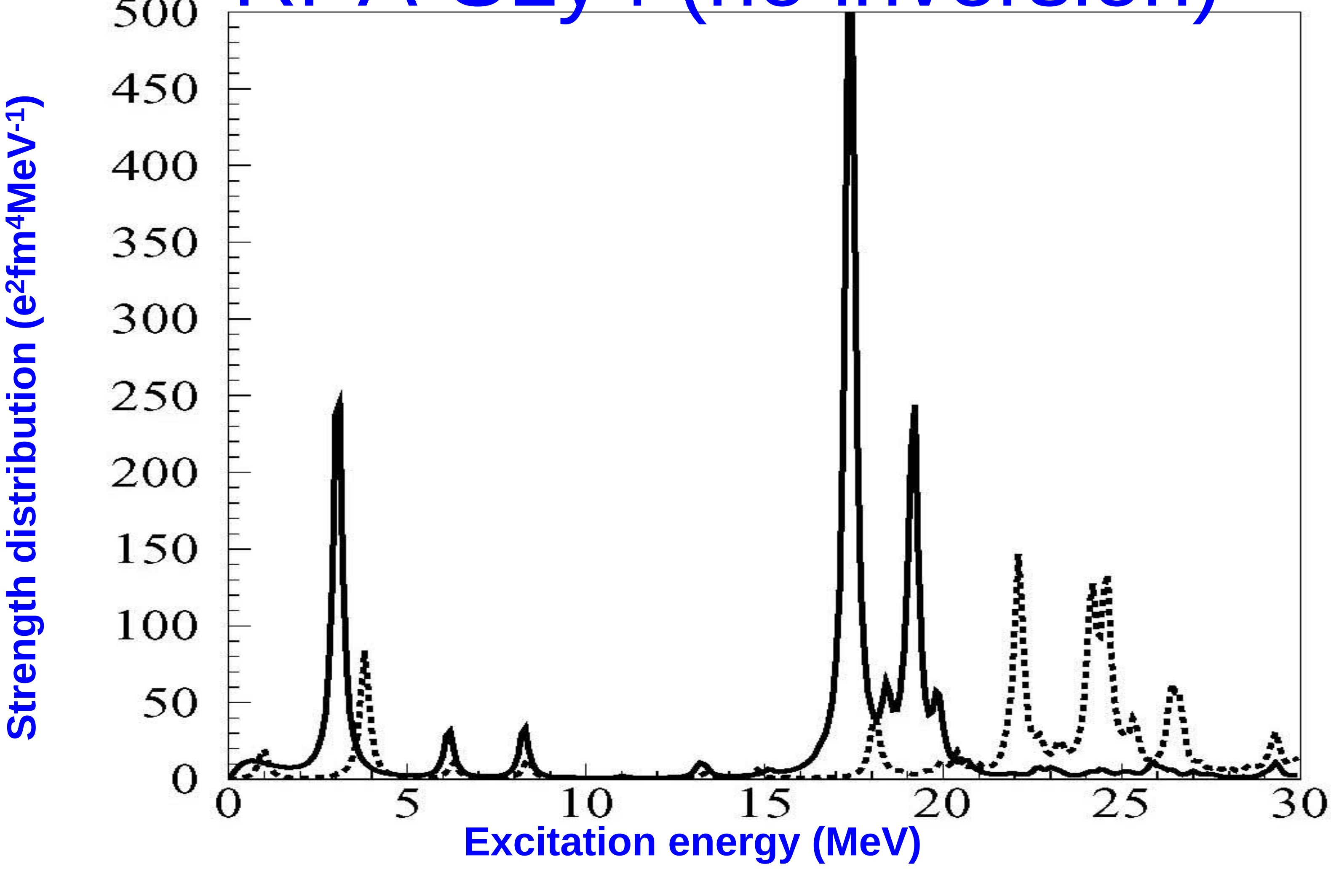
Quadrupole excitations in ^{46}Ar RPA-SLy4 (no inversion)



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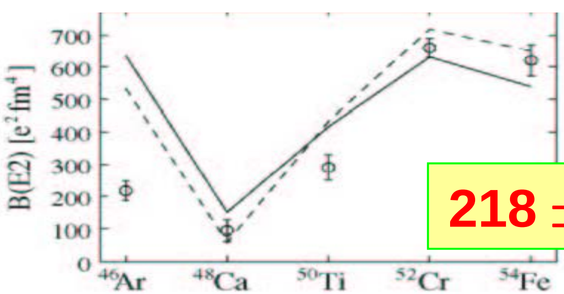
$B(E2;0^+_{g.s.} \rightarrow 2_1^+)$ ($e^2 fm^4$)

	SkI5	SLy4
B (E2) (e^2 fm^4)	256	24



218 ± 31 e^2 fm^4

FIG. 5. Ratios of M_n/M_p to N/Z (top panel) and $B(E2;0^+_{g.s.} \rightarrow 2^+_1)$ values [14] (bottom panel) for the 2^+_1 states of even-even $N=28$ isotones. The value of $(M_n/M_p)/(N/Z)$ for ^{46}Ar (•) is from the present work, and those for $A=48-54$ (○) are derived from deformation lengths given in Refs. [14,33,42]. The solid and dashed lines are shell model calculations described in the text.

Khan, Grasso, Margueron, Van Giai, in press NPA

Riley, et al. PRC 72, 024311 (2005)

Raman, et al., At. Data Nucl. Data Tables 36, 1 (2001)

Pairing?

HFB + QRPA

*** Ring and Schuck, The Nuclear Many-Body Problem, Springer-Verlag, Berlin, 1980**

*** de Gennes, Superconductivity of Metals and Alloys, Benjamin, New York, 1966**

*** Dobaczewski, Flocard, and Treiner, NPA 422 (1984) 103**

•Grasso, Sandulescu, Van Giai, and Liotta, PRC 64 (2001) 064321

•Khan, Sandulescu, Grasso, and Giai, PRC 66, 024309 (2002)

Pairing interaction. Choice of parameters

$$V(\vec{r}_1 - \vec{r}_2) = V_0\left[1 - x\left(\frac{\rho(r)}{\rho_0}\right)^{\gamma}\right]\delta(\vec{r}_1 - \vec{r}_2)$$

ρ_0 = 0.16 fm^{-3}

x = 0.5

γ = 1

Ecutoff = 70 MeV

V_0 is chosen to reproduce the two-proton separation energy

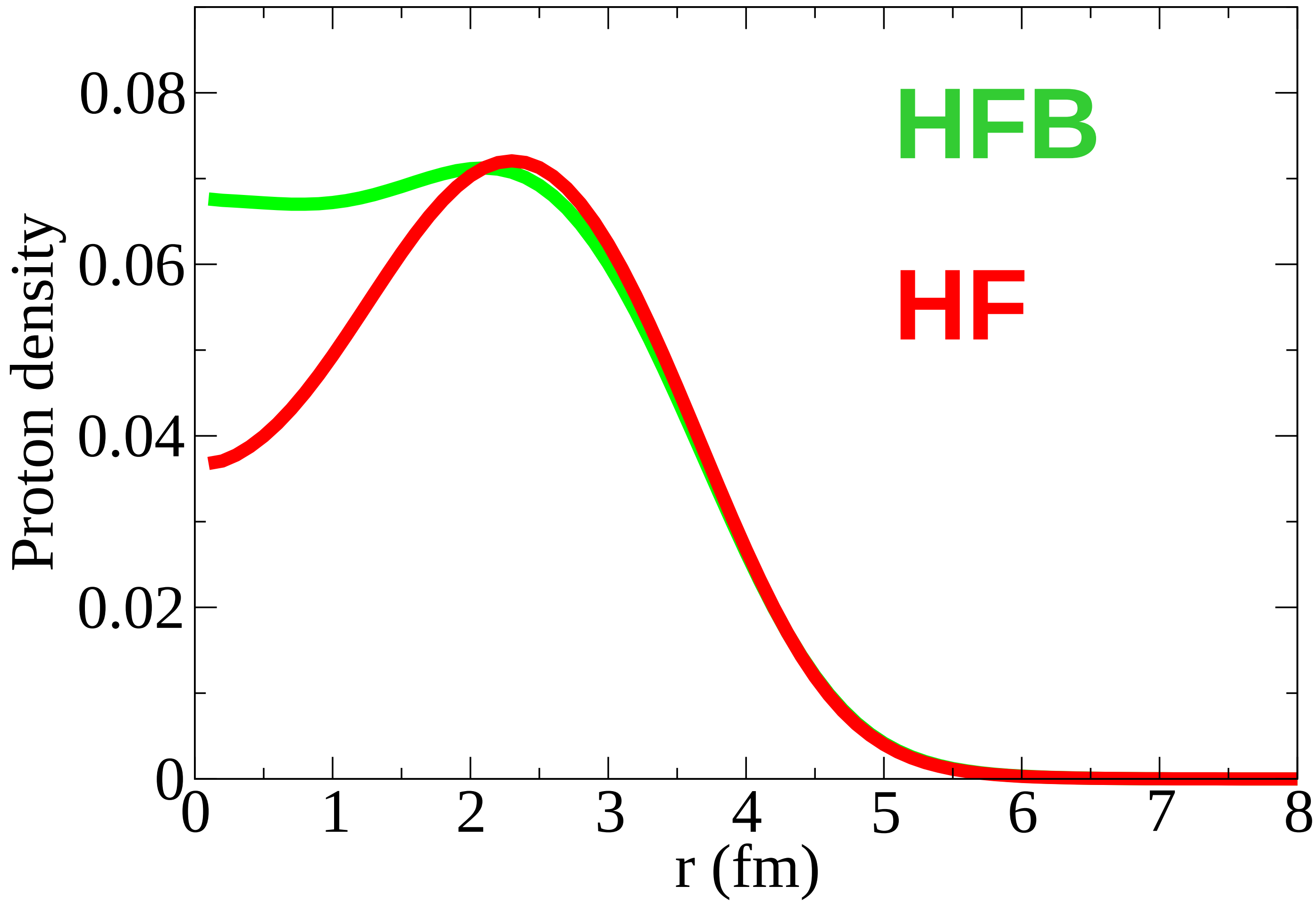
(S_{2p} exp. = 33.42 MeV)

Results with pairing

- **HFB. According to the occupation of the proton s state (intensity of the pairing interaction), the bubble structure is partially washed out and, as a consequence, the reduction of the neutron p splitting in ^{46}Ar is weaker**

- **QRPA results for the low-lying 2^+ state are comparable to SLy4-RPA results (no inversion)**

Proton density in ^{46}Ar with SkI5-



Occupation of the state 2s1/2 : 0.54 -> neutron p splitting is much less reduced (19%)

In very neutron-rich Ar isotopes (near the neutron drip line) the gap between the s and d states is larger.

When pairing is taken into account, the bubble structure still remains

Conclusions

- **Evolution of proton states in Ca isotopes. Inversion between s and d states in ^{48}Ca and in very neutron-rich isotopes (HF and RMF)**
- **Analysis with the equivalent potential: central term (U_0) depends on the filling of neutron orbitals**
- **Effect of the tensor contribution**
- **Ar isotopes: inversion and bubbles?**
- **Quadrupole excitations. B(E2) in ^{46}Ar**
- **Pairing**
- **Beyond mean field: particle-phonon coupling**