

Theory Network for Nuclear Structure and Reactions

**Extracting spectroscopic information from transfer reactions
involving weakly bound nuclei**

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Planned content:

1. Spectroscopy to unbound states:

- the ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

2. Spectroscopy to bound states:

- the ${}^{10}\text{Be}(d,p){}^{11}\text{Be}$ case

Motivation

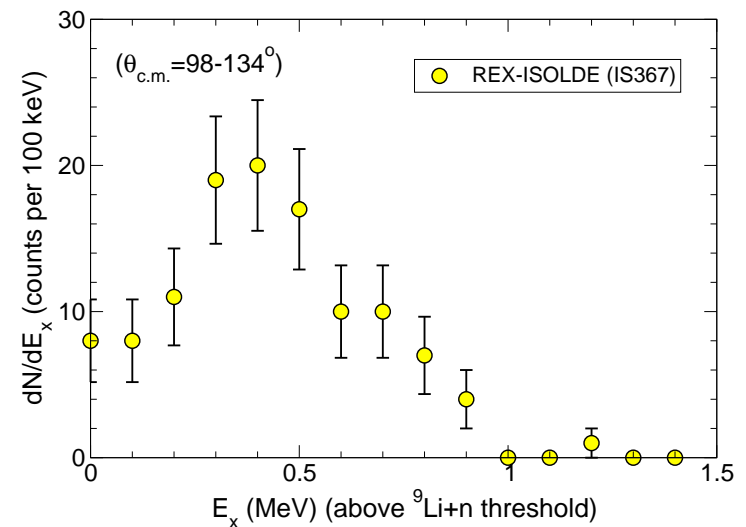
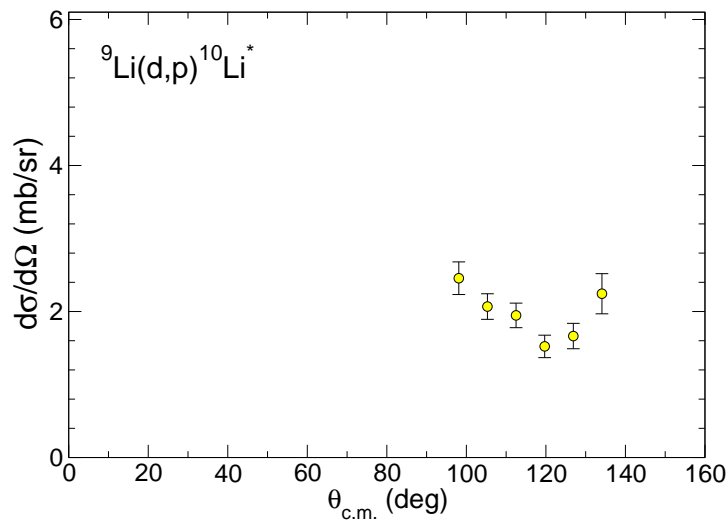
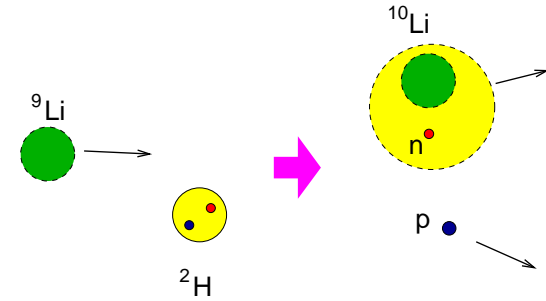
- Many experiments planned in the new experimental facilities involve transfer of weakly bound systems, including transfer to unbound states
Eg: LOI for Spiral 2: *Unbound states of neutron-rich isotopes via direct reactions*
- The standard tool to analyse transfer reactions, ie, DWBA, might be inappropriate because it ignores coupling to unbound states

Part I: Spectroscopy to unbound states: the ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

The Experiment:

- REX-ISOLDE (2002)
- ${}^9\text{Li}$ beam on D target at $E = 2.75$ MeV/u
- The experiment provided angular and energy distributions for protons $\Rightarrow {}^{10}\text{Li}$.

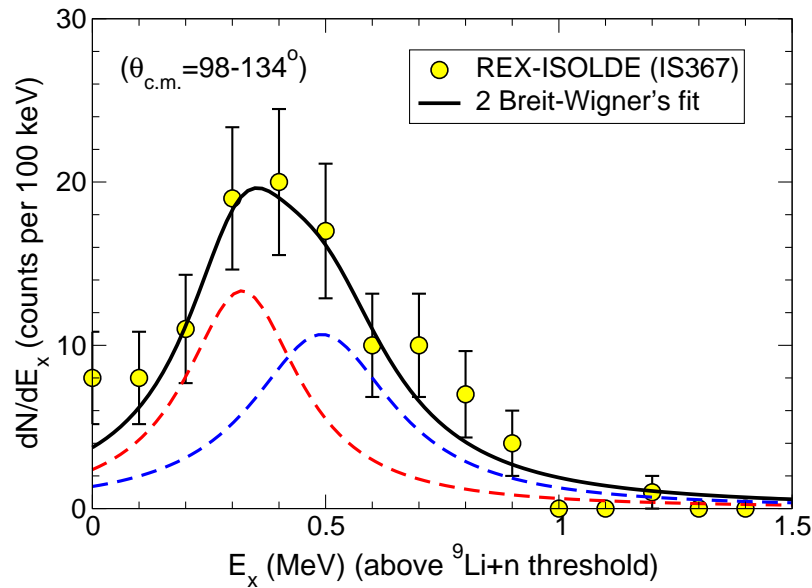


Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

$$\frac{d\sigma}{dE_x} \propto \frac{\Gamma}{(E - E_r)^2 + \frac{1}{4}\Gamma^2}$$

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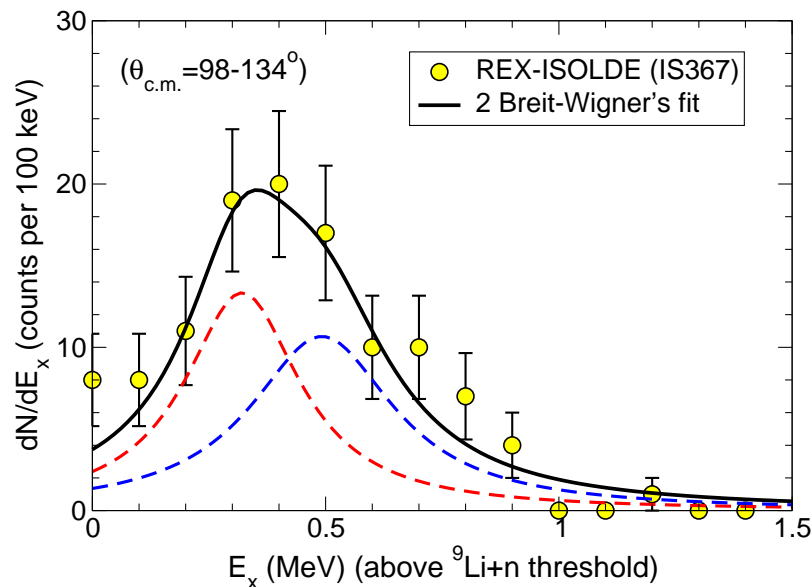


$$E_r^{(1)} = 0.32 \text{ MeV} \quad \Gamma^{(1)} = 0.30 \text{ MeV}$$

$$E_r^{(2)} = 0.49 \text{ MeV} \quad \Gamma^{(2)} = 0.37 \text{ MeV}$$

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This procedure is questionable, because (i) very ambiguous and (ii) ignores the effect of the reaction on the observed shapes

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

MOTIVATION: *What spectroscopic information can be obtained from the ${}^{10}\text{Li}$ distributions?*

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IMPORTANT QUESTIONS TO ADDRESS:

- **Q:** What is the mechanism producing protons?
A: Direct (compound estimated to be very small)

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A: Direct (compound estimated to be very small)

- **Q:** How does the reaction mechanism affect the ${}^{10}\text{Li}$ energy spectrum? (eg. relationship between observed bump and continuum structures)

A: Requires a proper reaction calculation; not just a fit to data

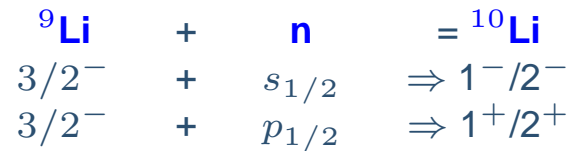
Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

${}^{10}\text{Li}$



$p_{3/2}$: bound state ($\epsilon_b \simeq 4.1$ MeV)

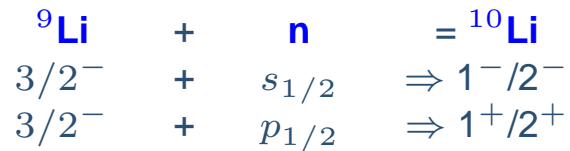
$s_{1/2}$: deeply 0s bound state
virtual 1s virtual state

$p_{1/2}$: low energy resonance

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

${}^{10}\text{Li}$

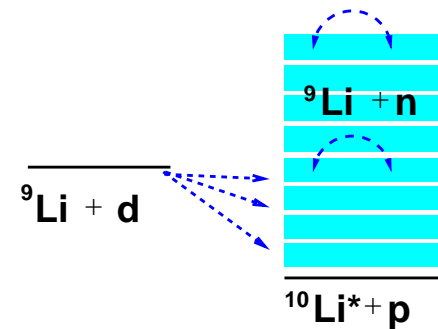


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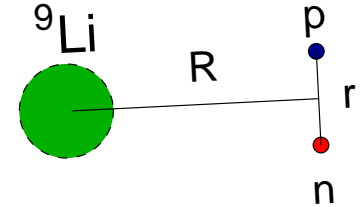
Transfer to the continuum (direct mechanism)



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Scattering amplitude:

$$T = \langle \Psi_f^{(-)} | V_{n+{}^9\text{Li}} + U_{p+{}^9\text{Li}} - U_\alpha | \chi_d^{(+)} \phi_d \rangle$$



- $\Psi_f^{(-)} \approx \Psi_f^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) = \sum_i \chi_{p-{}^{10}\text{Li}}^i(\mathbf{R}) \phi_{{}^{10}\text{Li}}^i(\mathbf{r})$
- $U_\alpha(\mathbf{R})$ taken to reproduce the elastic data \Rightarrow

Some features:

- No straight line (eikonal) assumption
- No zero-range approximation
- ${}^{10}\text{Li}$ structure naturally included in $\Psi_f^{(-)}$

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

${}^{10}\text{Li}$ structure: neutron- ${}^9\text{Li}$ core potential model: (Garrido *et al*, NPA700(2002)117)

$$V_{nc}(r) = \begin{cases} -V_{s12} \exp(-r^2/a^2) & (s_{1/2}) \\ -V_{p12} \exp(-r^2/a^2) & (p_{1/2}) \\ -V_{p32} \exp(-r^2/a^2) & (p_{3/2}) \end{cases}$$

- $a=2$ fm \Rightarrow correct rms for ${}^9\text{Li}$

Two free parameters: V_{s12} , V_{p12}

- $V_{s12} \leftrightarrow E_s \leftrightarrow a_s$
- $V_{p12} \leftrightarrow E_{res} \leftrightarrow \Gamma$

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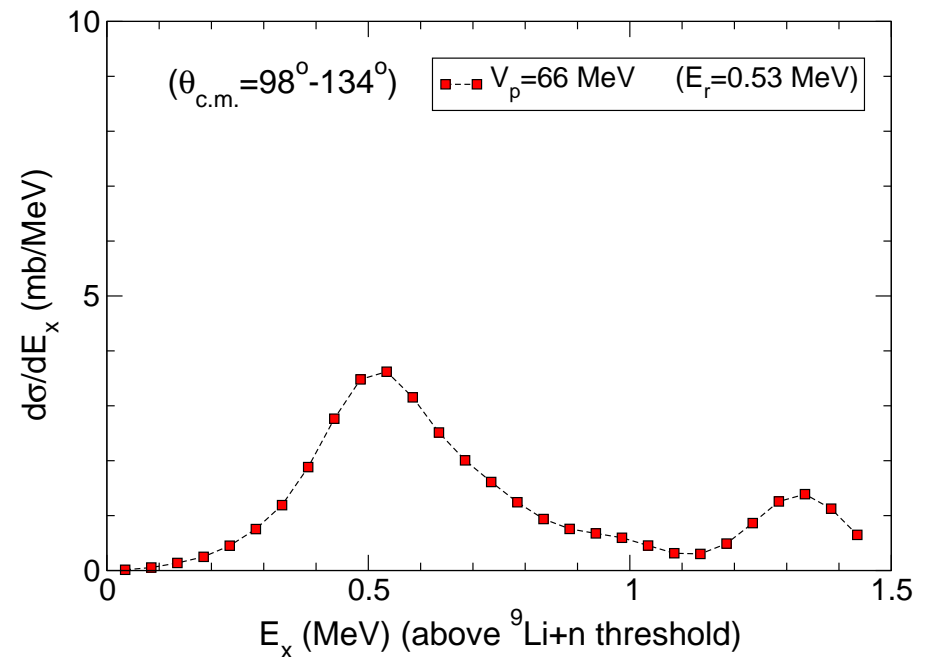
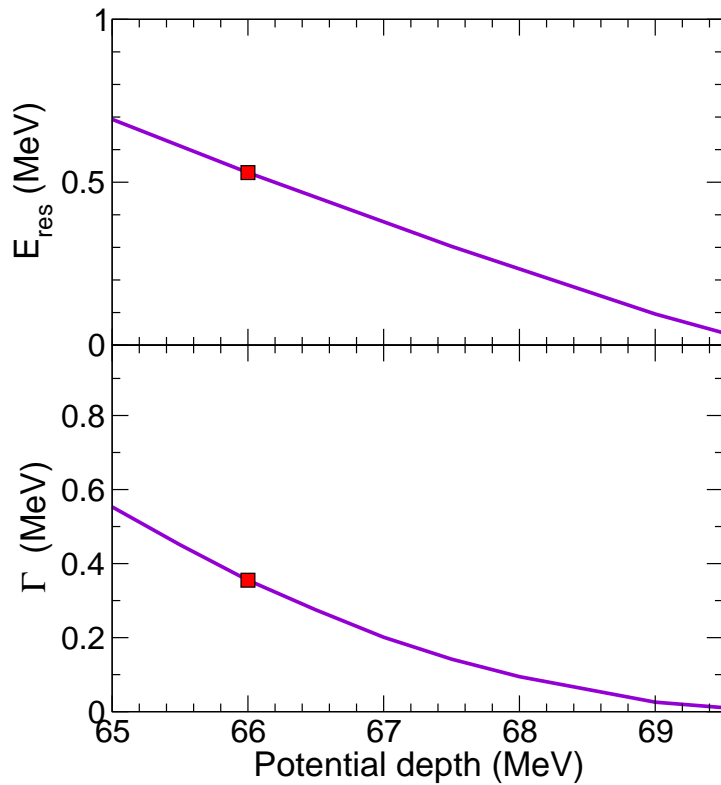
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No spin-spin interaction $\Rightarrow 1^+/2^+$ and $1^-/2^-$ degenerated

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure: $p_{1/2}$ resonance

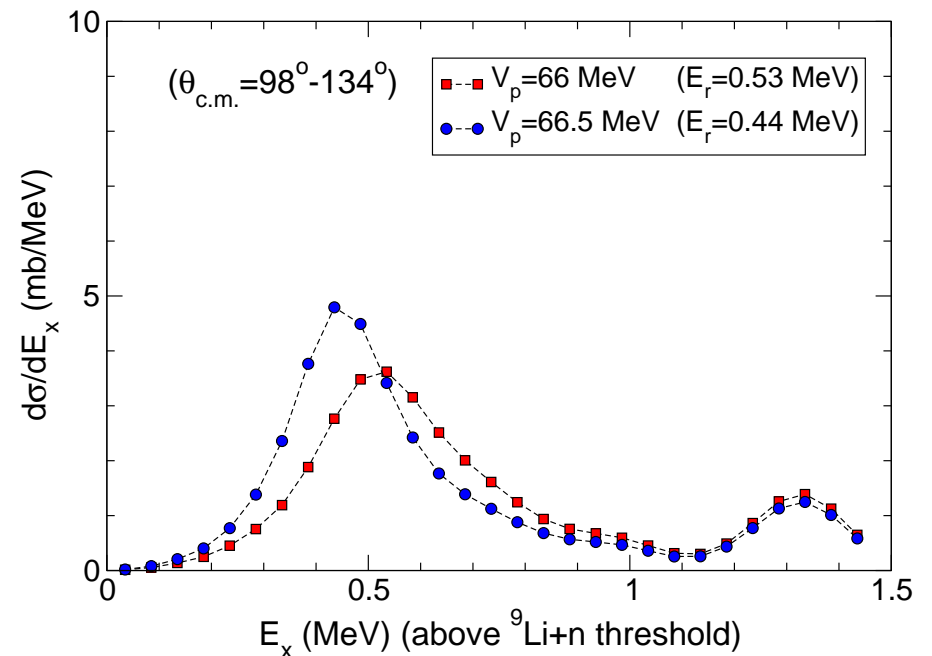
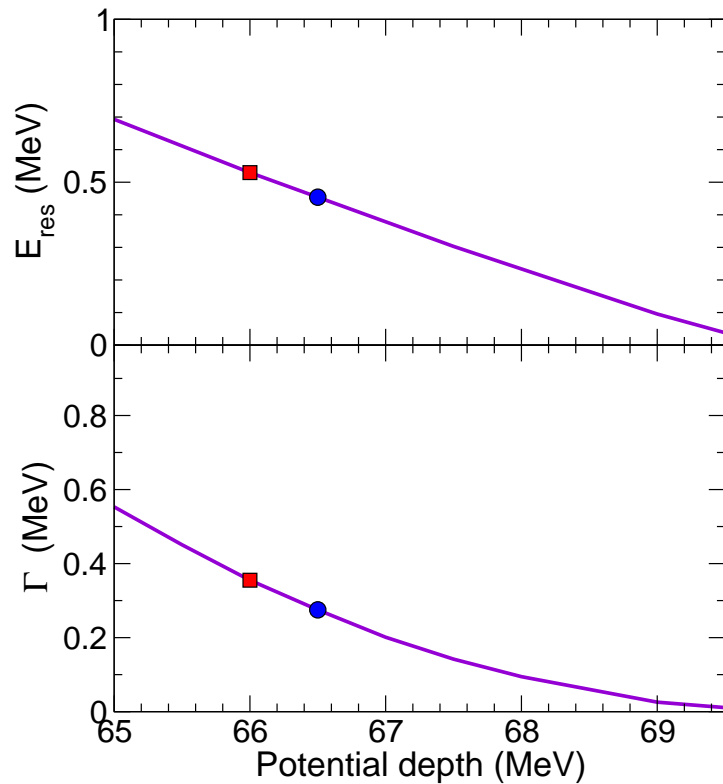
Reaction



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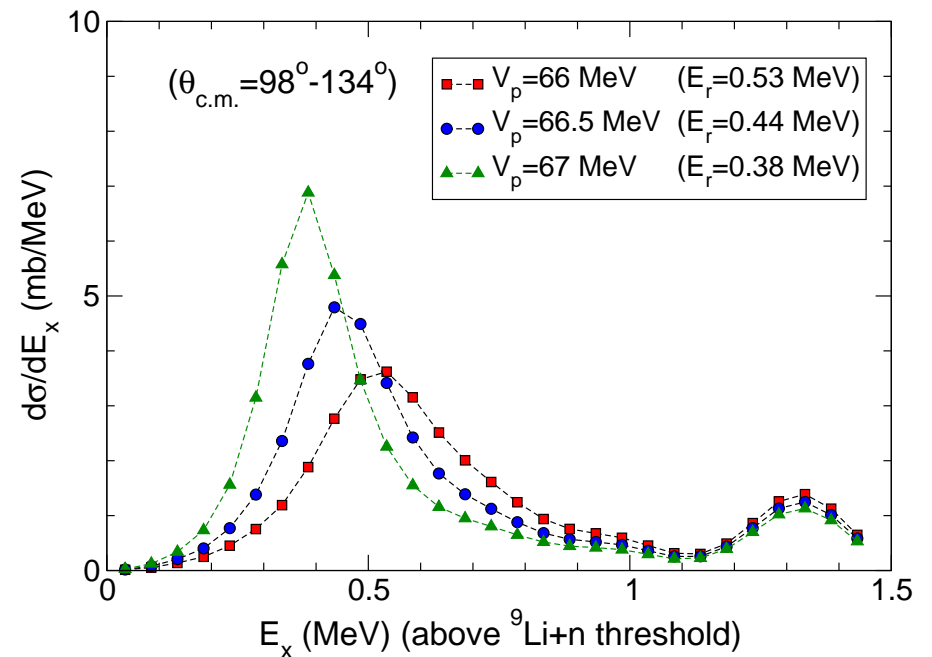
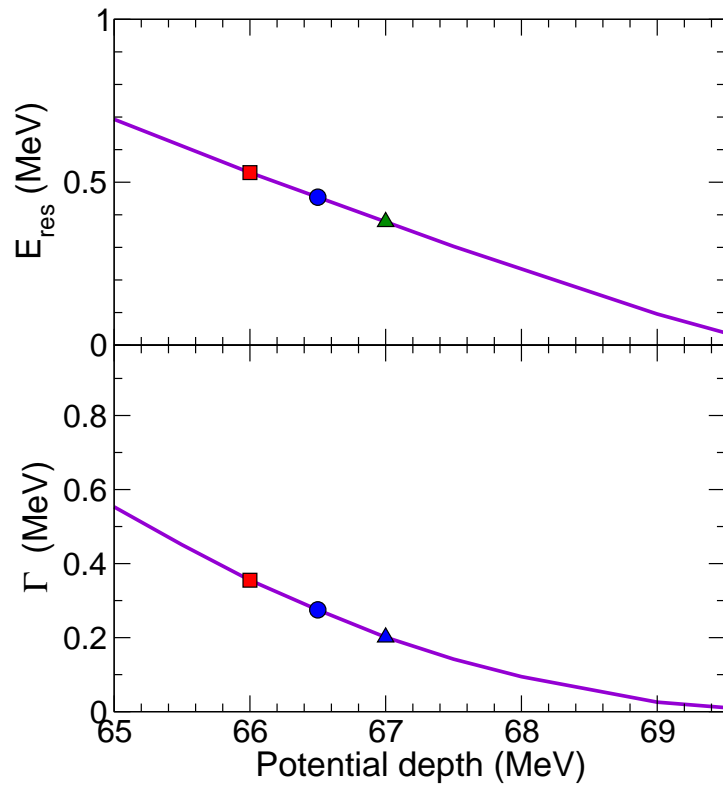
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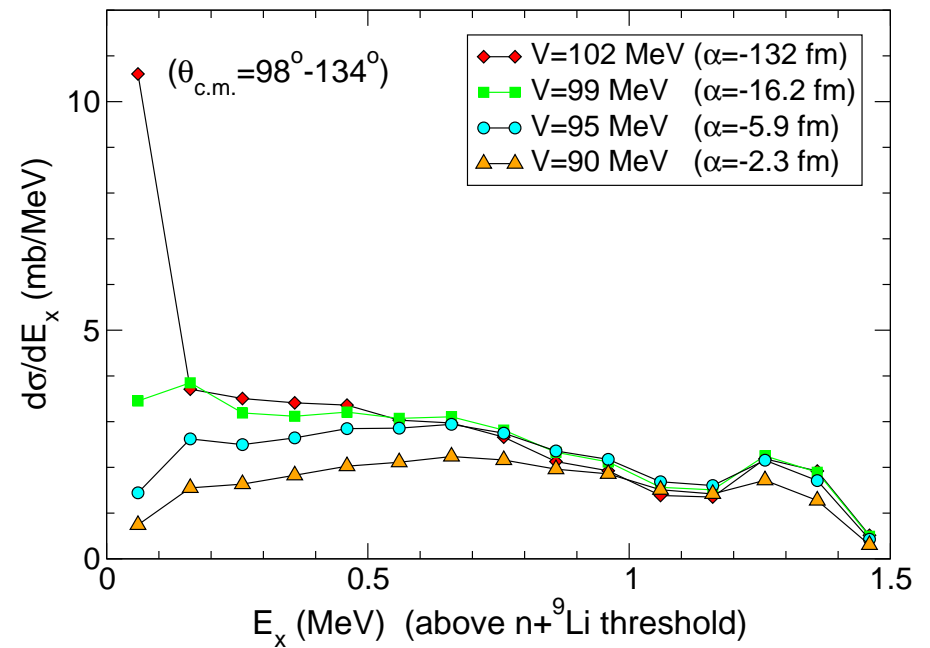
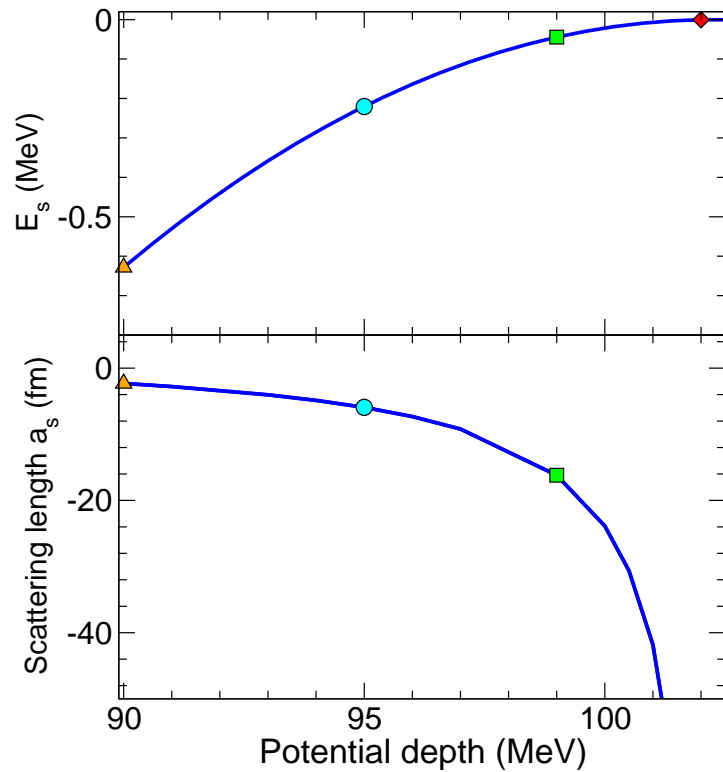
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Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure: $s_{1/2}$ v.s.

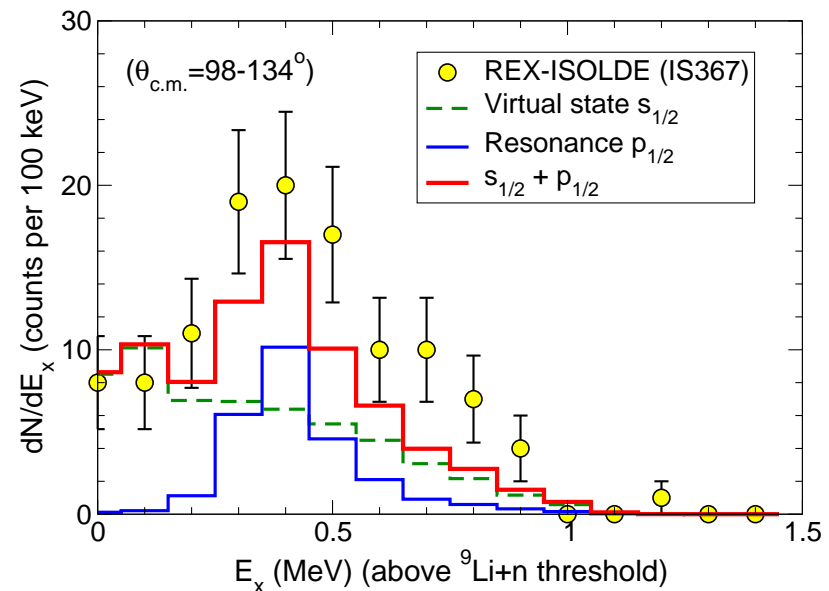
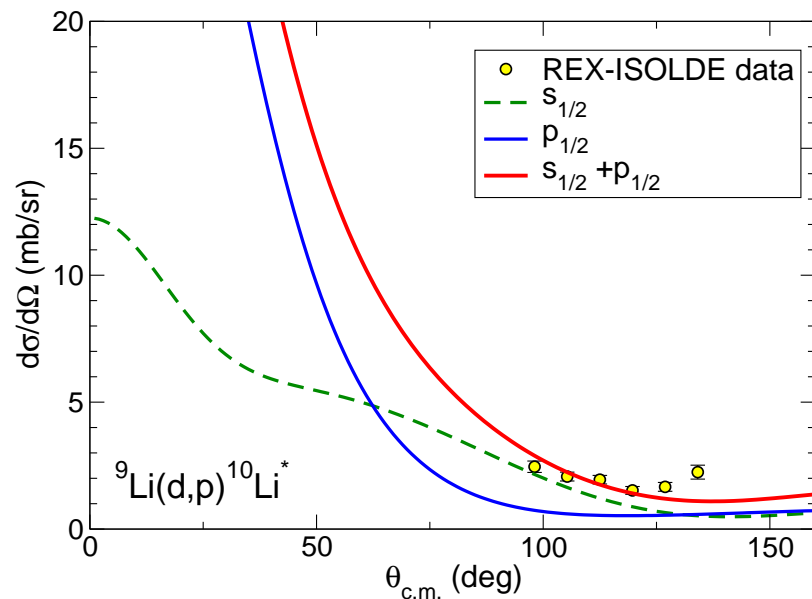
Reaction



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$

BEST FIT RESULTS: HP.Jeppesen et al, PLB642 (2006) 449

- $p_{1/2}$ resonance ($1^{+}/2^{+}$ doublet): $E_r \simeq 0.38$ MeV, $\Gamma = 0.2$ MeV
- $s_{1/2}$ virtual state ($1^{-}/2^{-}$ doublet): $a_s \simeq -24$ fm



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

	Simon <i>et al</i> (1)	${}^9\text{Li}(d,p){}^{10}\text{Li}$
Virtual state	$a_s = -30_{-31}^{+12}$	$a_s \simeq -24$ fm
p -resonance	$E_r = 0.510(44)$ $\Gamma = 0.54(16)$	$E_r = 0.38$ MeV $\Gamma = 0.20$ MeV
d -resonance	$E_r = 1.486(88)$ $\Gamma < 2.2$	- -

(1) Simon *et al*, NPA791 (2007) 267,
based on the analysis of ${}^{11}\text{Li}$ fragmentation on ${}^{12}\text{C}$ at 264 MeV/u.

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Some conclusions from the analysis of ${}^9\text{Li}(d,p){}^{10}\text{Li}$ data:

- The ${}^9\text{Li}(d,p){}^{10}\text{Li}$ has proved to be a useful tool to extract information of ${}^{10}\text{Li}$.
- Using the transfer to the continuum approach, combined with a simple inert-core model for ${}^{10}\text{Li}$, is a reliable method to determine the parameters of these continuum structures.

Open problems and further work

- Separation of the $1^+/2^+$ and $1^-/2^-$ doublets \Rightarrow better energy resolution?
- No evidence of d -wave resonance below $E_x < 1$ MeV, but could appear at higher energies \Rightarrow would require better acceptance at these energies and/or higher beam energies.
- What is the role of core excitation (${}^9\text{Li}^*$) and how can it be incorporated?.

Part II: Spectroscopy to bound states: the $^{10}\text{Be}(d,p)^{11}\text{Be}$ case

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Motivation and antecedents:

- The ^{11}Be g.s. is long known to consist mainly in a combination of two components:
(i) $^{10}\text{Be}(\text{g.s.}) \otimes 2s_{1/2}$ and (ii) $^{10}\text{Be}(2^+) \otimes 1d_{5/2}$
- In *Phys. Rev. C* **59 (1999)1545**, N.K. Timofeyuk and R.C. Johnson (TJ) analysed the $^{10}\text{Be}(d,p)^{11}\text{Be}$ data in order to determine the weight of the $2s$ component, leading to the value $S_f \simeq 0.44$.
- This value is significantly smaller than most shell model predictions!

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

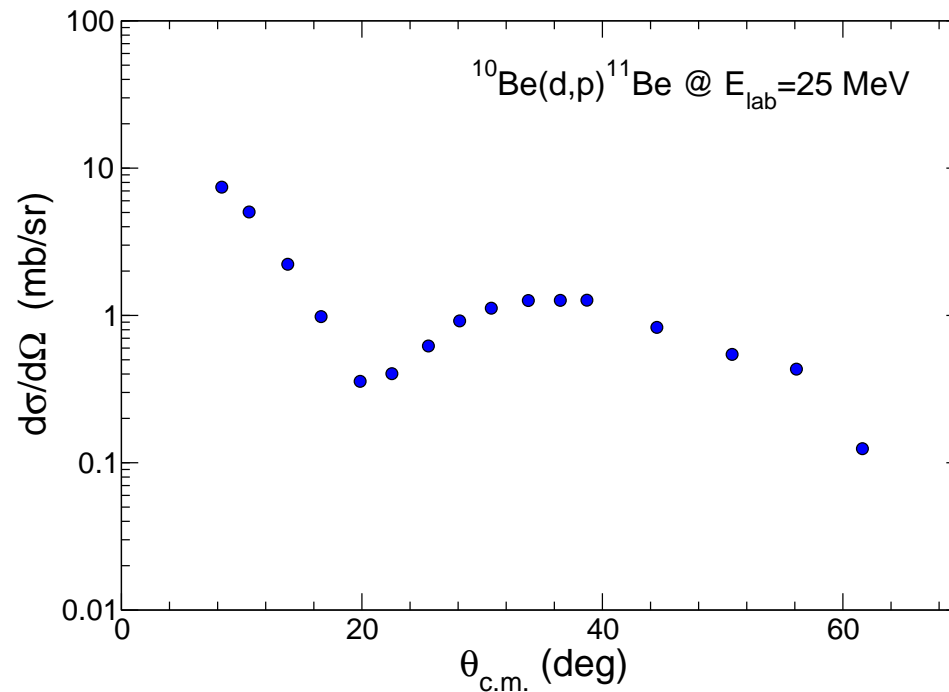
Comparison of TJ spectroscopic factor with structure calculations:

	$S(0^+ \otimes 2s)$	$S(2^+ \otimes 1d)$
Analysis of $^{10}\text{Be}(d,p)^{11}\text{Be}$ [1]	0.44	-
PVM (Bhattacharya & Krishan) [3]	0.70	-
CC rotational coupling (Nunes <i>et al.</i> [4])	0.78	0.20
Shell-model (Warburton & Brown [5])	0.74	0.19
Vibrational coupling (Vinh-Mau [6])	0.80	0.20
Generator-Coord. (Descouvemont [8])	0.92	0.07

1. N.K. Timofeyuk and R.C. Johnson, Phys.Rev. C59 (1999)1545.
2. T. Bhattacharya and K. Krishan, Phys. Rev. C 56 (1997) 212.
3. F.M. Nunes, I.J. Thompson and R.C. Johnson, Nucl. Phys. A 596 (1996) 171.
4. E.K. Warburton and B.A. Brown, Phys. Rev. C 46 (1992) 923.
5. N. Vinh Mau, Nucl. Phys. A 592 (1995) 33 N. Vinh Mau and J.C. Pacheco, Nucl. Phys. A 607 (1996) 163.
6. P. Descouvemont, Nucl. Phys. A 615 (1997) 261

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Experimental data: $^{10}\text{Be}(d,p)^{11}\text{Be}_{\text{gs}}$

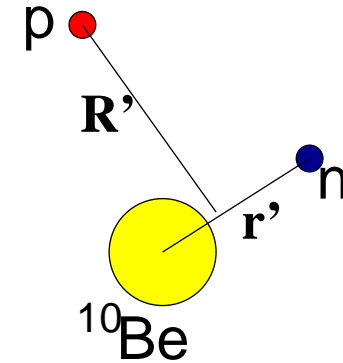


Zwieglinski et al, NPA315 (1979) 124

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Exact expression for the transition amplitude (post form):

$$T^{\text{exact}} = \langle \Phi_{\beta}^{(-)} | V_{pn} + U_{p-^{10}\text{Be}} - U_{\beta} | \Psi_d^{(+)} \rangle$$



- $\Psi_d^{(+)}$: exact 3-body WF
- $\Phi_{\beta}^{(-)}$: 3-body WF, obtained as solution of the equation:

$$[E - i\epsilon - K - V_{n-^{10}\text{Be}} - U_{\beta}] \Phi_{\beta}^{(-)}(\mathbf{r}', \mathbf{R}') = 0$$

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Timofeyuk-Johnson amplitude:

In the exact expression: $U_\beta \equiv U_{p-^{10}\text{Be}}(\mathbf{r}_{p-^{10}\text{Be}})$

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_\beta^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

- $[E - i\epsilon - K - V_{n-^{10}\text{Be}} - U_{p-^{10}\text{Be}}] \tilde{\Phi}_\beta^{(-)}(\mathbf{r}', \mathbf{R}') = 0$

Advantages:

- Only binary interactions are needed (n - p , p - ^{10}Be and n - ^{10}Be)
- $\Psi_d^{(+)}(\mathbf{r}, \mathbf{R})$ is only required within for small n - p separations ($r \approx 0$)

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Evaluation of the TJ amplitude in the adiabatic approximation: Phys.Rev. C59, 1545 (1999)

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_{\beta}^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

If $\epsilon_x \ll E_{\text{c.m.}}$:

- $\tilde{\Phi}_{\beta}^{(-)} \approx \tilde{\Phi}_{\beta}^{ad} = \chi_{p-^{10}\text{Be}}(\mathbf{r}_{p-^{10}\text{Be}}) \phi_{^{11}\text{Be}}(\mathbf{r}') e^{-i\alpha \mathbf{k}_{\beta} \mathbf{r}'}$

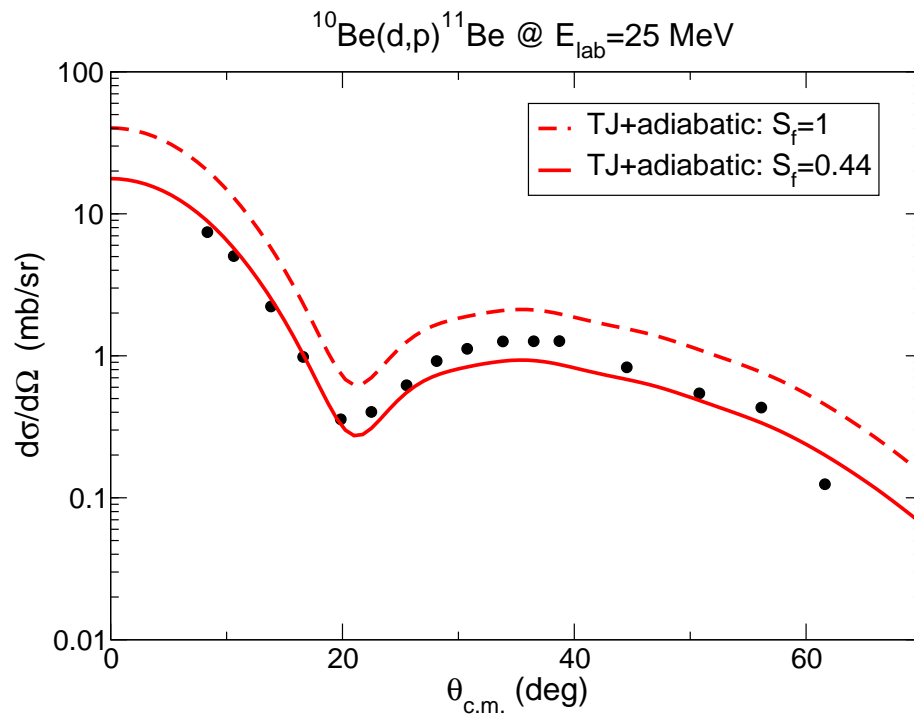
(Johnson, Al-Khalili and Tostevin, PRL79 (1997) 2771)

- $\Psi_d^{(+)} \approx \chi_d^{JS}(\mathbf{R}) \phi_d(\mathbf{r})$

($\chi_d^{JS}(\mathbf{R})$ calculated with the Johnson-Soper potential)

(Johnson and Soper, PRC1 (1979) 976)

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$



- good agreement with the data, but
- leads to a very small spectroscopic factor ($S_f \simeq 0.44$)

Timofeyuk and Johnson, Phys.Rev. C59, 1545 (1999)

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

The anomalously small spectroscopic factor extracted with the TJ expression can be due to many reasons:

1. The approximations involved in the evaluation of the TJ amplitude
Is it possible to make a more accurate evaluation?
2. The inadequacy of the TJ amplitude
Does the “standard” amplitude give to consistent results?
3. Simplified model for ^{11}Be (core excitation is ignored)
What is the effect of core excitation?
4. Incorrect normalization of the data
If everything fails... is it worth to propose a new experiment?
5. ...

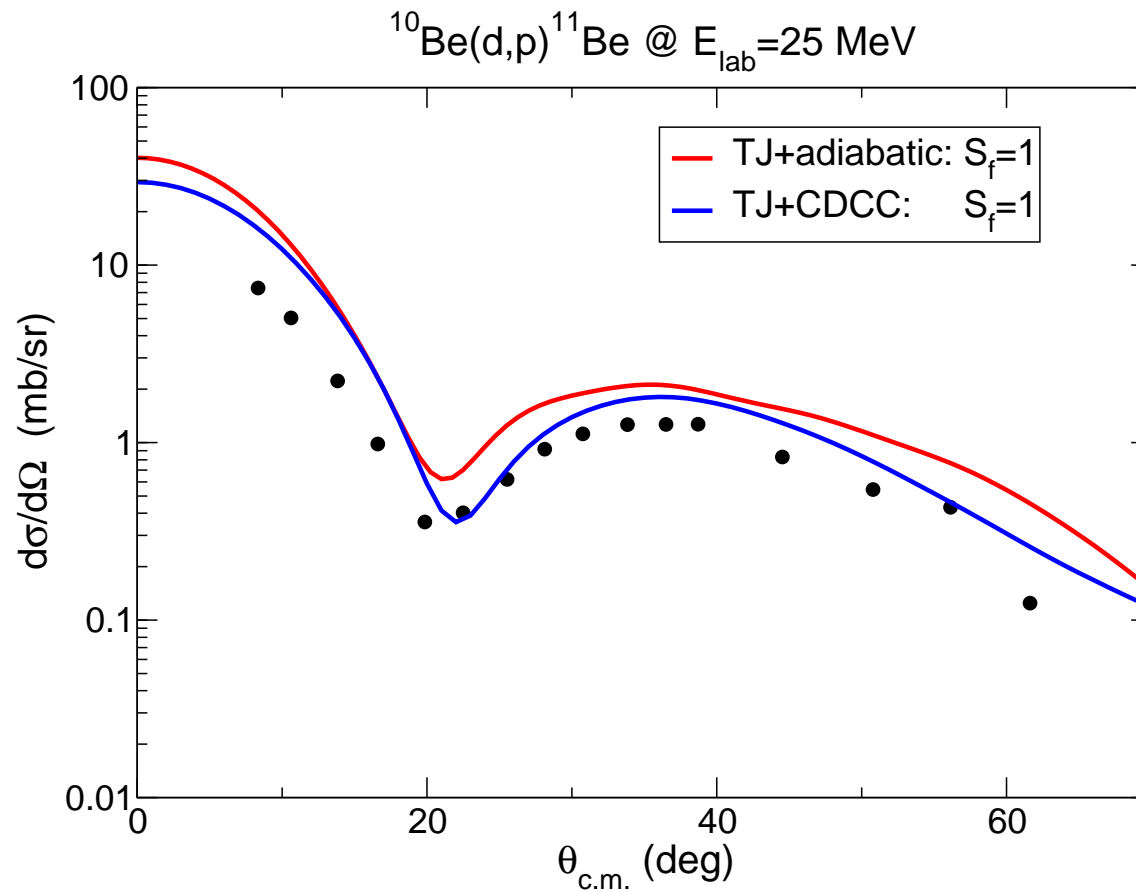
Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Evaluation of the TJ amplitude using CDCC wfs:

$$T_{\text{post}}^{\text{TJ}} = \langle \tilde{\Phi}_{\beta}^{(-)} | V_{pn} | \Psi_d^{(+)} \rangle$$

- $\tilde{\Phi}_{\beta}^{(-)}$ and $\Psi_d^{(+)}$ are approximated by CDCC expansions:
 - ◆ $\Psi_d^{(+)}$ expanded in p-n states
 - ◆ $\tilde{\Phi}_{\beta}^{(-)}$ expanded in n- ^{10}Be states
- No adiabatic approximation is involved

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$



Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Calculations with the “standard” transition amplitude:

$$T^{\text{exact}} = \langle \Phi_{\beta}^{(-)} | V_{pn} + U_{p-^{10}\text{Be}} - U_{\beta} | \Psi_d^{(+)} \rangle$$

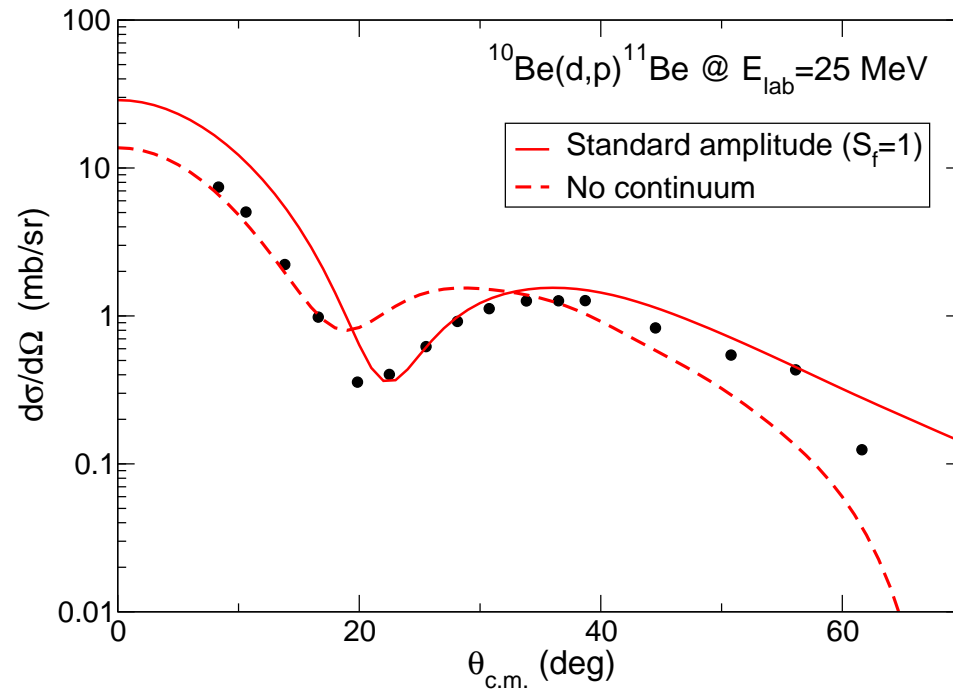
Evaluation within the CDCC approximation:

- $\Psi_d^{(+)} \approx \Psi_d^{\text{CDCC}} = \sum_i \chi_d^i(\mathbf{R}) \phi_d^i(\mathbf{r})$
- $U_{\beta} = \langle \phi_{^{11}\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} | \phi_{^{11}\text{Be}} \rangle \equiv U_{00}(R')$

$$T_{\text{post}}^{\text{CDCC}} = \langle \chi_p \phi_{^{11}\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} - U_{00} | \Psi_d^{\text{CDCC}} \rangle$$

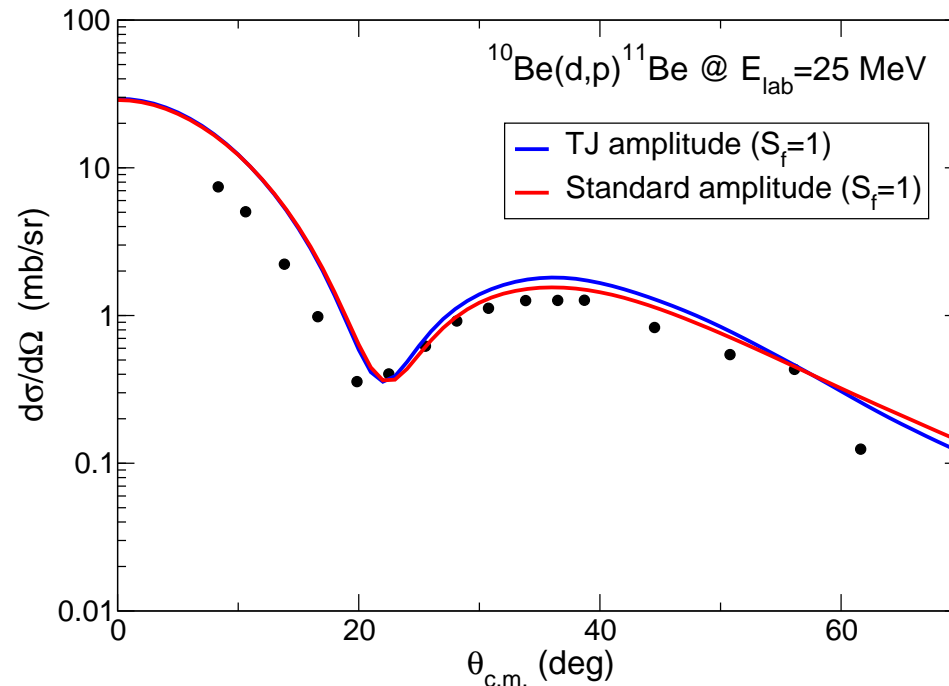
Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

“Standard” amplitude with CDCC wfs:



Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

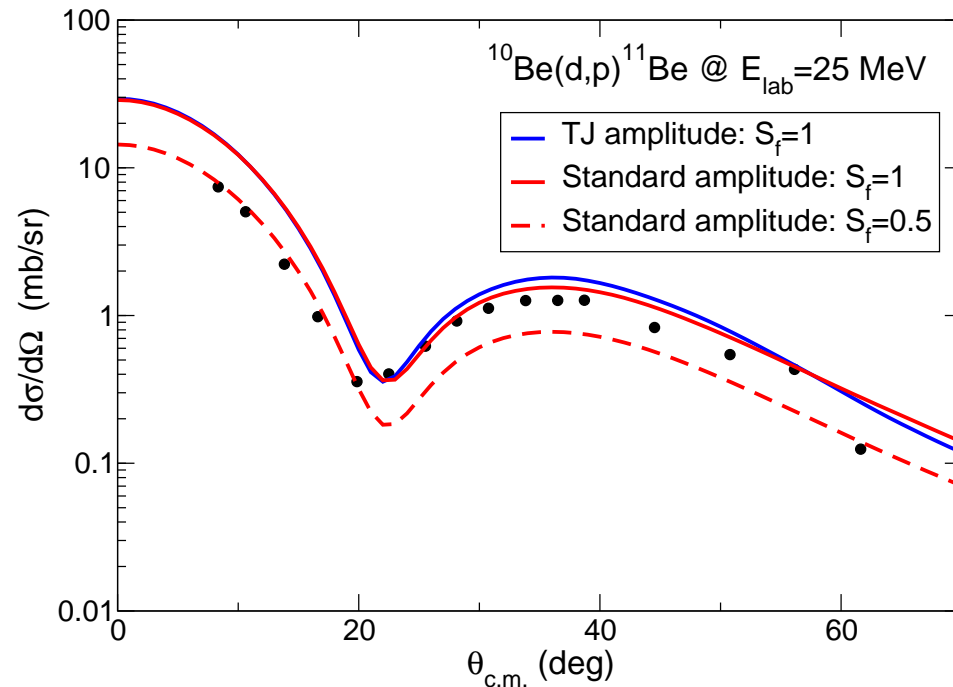
Comparison of the TJ and “standard” amplitudes with CDCC wfs



The TJ and “standard” amplitudes provide consistent structure information (ie, the same S_f for $^{11}\text{Be}_{\text{gs}} \rightarrow ^{10}\text{Be}_{\text{gs}} + 1n$)

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Comparison of the TJ and “standard” amplitudes with CDCC wfs



If S_f is adjusted to reproduce the data at forward angles $\Rightarrow S_f \simeq 0.5$

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Conclusions:

- The reaction $^{10}\text{Be}(d,p)^{11}\text{Be}$ has been re-analysed using three different scattering frameworks:
 1. TJ expression within the adiabatic approximation
 2. TJ expression with CDCC wfs (non-adiabatic)
 3. “Standard” expression with CDCC wfs
- These approaches confirm the relevance of the coupling to the continuum and give rise a similar spectroscopic factor for ^{11}Be ($S_f \approx 0.44 - 0.5$).
- The small value for S_f remains an open problem. Some improvements are underway but suggestions are welcome:
 - ❖ Perform a more accurate evaluation of the transition amplitude? (Faddeev?).
 - ❖ Improve the description of structure model (eg., core excitation)?

List of collaborators

- ${}^9\text{Li}(d,p){}^{10}\text{Li}$ analysis:
 - ❖ Joaquín Gómez Camacho
 - ❖ Karsten Riisager, Henrik Jeppesen
(for the REX-ISOLDE Collaboration)
- ${}^{10}\text{Be}(d,p){}^{11}\text{Be}$ analysis:
 - ❖ Ron Johnson and Filomena Nunes