TALENT: theory for exploring reaction experiments

Breakup reactions with the Continuum discretized coupled channel method

Filomena Nunes
standard 3-body CDCC in one slide

- three-body Hamiltonian for reaction
- three-body wavefunction expanded in terms of projectile states
- projectile described in terms of single particle states
- optical potentials from elastic scattering
How to do a breakup calculation with fresco?

cdc < short.in > fresco.in

Box B.4 Fresco input for the breakup of $^8$B on $^{208}$Pb at 82 MeV per nucleon
(short version)
How to do a breakup calculation with fresco?

8B+208Pb ; N+C breakup with q=0,1,2
CDCC
&CDCC
   hcm=0.01  rmatch=-60 rasym=1000 accrcy=0.001 absend=-50
   elab=656
   jbord= 0 200 300 1000 9000
   jump = 1 10 50 200
   thmax=20 thinc=0.05 cutr=-20 smats=2 xstabl=1
   ncoul=0 reor=0 q=2
/

**CDCC namelist:** contains all the relevant parameters for the fresco namelist
- **rmatch<0** and **rasym** (couplings have long range so match to coupled channel Coulomb function all the way to rasym)
- **Jmax=9000!** Need many partial waves for this high energy. But jumping...(all j’s up to 200, then in 10’s up to 300, then in 50’s up to 1000, then in 200’s up to 9000)
- Theta grid: angular distribution very forward focused – high energy
- **cutr** – avoids integration at small r due to the strong repulsive potentials (cutr=-20fm put the integration starting point 20 fm inside the coulomb turning radius)
How to do a breakup calculation with fresco?

8B+208Pb ; N+C breakup with q=0,1,2
CDCC
&CDCC
  hcm=0.01  rmatch=-60 rasy=1000 accrcy=0.001 absend=-50
  elab=656
  jord= 0 200 300 1000 9000
  jum= 1 10 50 200
  thmax=20 thinc=0.05 cutr=-20 smats=2 xstabl=1
  ncoul=0 reor=0 q=2
/

Some key additional parameters:
• **ncoul** (0=nucl+Coul; 1=nucl only, 2=Coul only)
• **reor** (different choices for the <\phi_i |V| \phi_i> couplings)
• **q** (multipoles to include: all multipoles up to 2)
How to do a breakup calculation with fresco?

&NUCLEUS part='Proj' name='8B' charge=5 mass=8
   spin=1.5 parity=-1 be = 0.137 n=1 l=1 j=1.5 /
&NUCLEUS part='Core' name='7Be' charge=4 mass=7 /
&NUCLEUS part='Valence' name='proton' charge=1 mass=1 spin=0.5/
&NUCLEUS part='Target' name='208Pb' charge=82 mass=208 spin=0 /

&BIN spin=0.5 parity=+1 start=0.001 step=0.50 end=10. energy=F l=0 j=0.5/ &BIN /

NUCLEUS namelist contains info for partition, overlap and states
•part=(the particle we wish to define including charge, mass, spin and parity, binding energy and the quantum numbers for the state)

BIN namelist contains info for overlap part
•energy=F (grid in momentum) or T (grid in energy)
•start, step and end define the details of the grid
•spin, parity are the total spin and parity of the projectile bin with quantum numbers l,j
How to do a breakup calculation with fresco?

&POTENTIAL part='Proj' a1=1 rc=2.65 /
&POTENTIAL part='Core' a1=208 rc=1.3 v=114.2 vr0=1.286 a=0.853 w=9.44 wr0=1.739 aw=0.809
&POTENTIAL part='Valence' a1=208 rc=1.3 v=34.819 vr0=1.17 a=0.75 w=15.340 wr0=1.32 aw=0.601/
&POTENTIAL part='Gs' a1=1 rc=2.391 v=44.675 vr0=2.391 a=.48 vso=4.898 rso0=2.391 aso=0.48 /

POTENTIAL namelist is identical to POT namelist is standard input
- `part=proj` (projectile-target interaction) = `core` (core-target interaction)
- `part=valence` (valence-target interaction)
- `part=gs/bin` (core-valence interaction in g.s. or continuum)
How to do a breakup calculation with fresco?

```plaintext
cdc < short.in > fresco.in
```

This command produce the standard fresco input where all states/overlaps are explicitly defined.

isc=12 standard normalization for momentum bins

kind=3 projectile single-particle coupling
How to do a breakup calculation with fresco?

sumxen < fort.13 > file.xen

sumbins < fort.16 > file.xsum

Fig. B.3. Breakup of $^8\text{B}$ on $^{208}\text{Pb}$ at 82 MeV/u. Left: p–$^7\text{Be}$ relative energy distribution. Right: center-of-mass angular distribution. Both are obtained with the input of Box B.3.
Capture rate \(^{3}\text{He}(\alpha, \gamma)^{7}\text{Be}\)

Capture rates at astrophysical energies are hard to measure and thus rely on extrapolations to low energies.

Motivates alternative methods using inverse reaction \(^{7}\text{Be} + \gamma \rightarrow \alpha + ^{3}\text{He}\)

(i) Coulomb Dissociation method
(ii) Asymptotic Normalization Coefficient (ANC) method
Coulomb Dissociation method

Cross sections larger for inverse breakup reaction
Can relate breakup cross section to capture rate using semi-classical reaction theory

complications
• **Nuclear breakup**
• E2 contributions
• Final state interactions
  (continuum-continuum couplings)

Motivated experiment at NSCL : $^7\text{Be} + ^{208}\text{Pb} \rightarrow 100 \text{ MeV/nucleon}$

Summers @ NSCL2004
$\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \frac{C_{lj}^2}{b_{nlj}^2} \left( \frac{d\sigma}{d\Omega} \right)_{\text{theory}}^{nlj}$

Outside range of nuclear force, in asymptotic region

$I_{cvlj}^P (r) \rightarrow S_{nlj}^{1/2} \phi_{nlj} (r) \rightarrow C_{cvlj}^P \frac{W_{-\eta,l+\frac{1}{2}} (2kr)}{r}$

Motivated experiment at Texas A&M : $^{7}\text{Be} + ^{12}\text{C}$ @ 25 MeV/nucleon

$C_{cvlj}^P = S_{nlj}^{1/2} b_{nlj}$
Breakup of $^7\text{Be}$: one application in detail

- $^7\text{Be}$ treated as 2-body projectile
- 3-body Hamiltonian for reaction

\[ H = T_R + U_{cT} + U_{vT} + h_{vc} \]
\[ h_{vc} = T_r + V_{vc} \]

$\phi$ is $^7\text{Be}$ ($\alpha$+$^3\text{He}$) cluster wavefunction defined by potential $V_{vc}$

\[
h_{vc}\phi_n(r) = \varepsilon_n\phi_n(r) \quad \varepsilon_n < 0
\]
\[
h_{vc}\phi_{lj}(k,r) = \varepsilon_k\phi_{lj}(k,r) \quad \varepsilon_k > 0
\]

$l = $ core-valence relative angular momentum

$j = $ projectile total angular momentum

\[ \text{fixed by} \]
\[ \text{binding energy for bound states} \]
\[ \text{resonances and scattering phase shifts for continuum} \]
Breakup of $^7$Be: continuum discretization

- Discretize continuum into bins
- Average wavefunction over a bin with weight $w_i(k)$

$$\phi_{i,lj}(r) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} w_i(k) \phi_{lj}(k, r) dk$$

$w_i(k)$ chosen so that the bin wavefunctions are real and normalized correctly using

$$N_i = \int_{k_{i-1}}^{k_i} |w_i(k)|^2 \, dk$$

- Label the quantum numbers for each bin by $\alpha \equiv i,lj$
- Now have 72 coupled channels

bound states $\rightarrow$

- $s_{1/2}$
- $p_{1/2}$
- $p_{3/2}$
- $d_{3/2}$
- $d_{5/2}$
- $f_{5/2}$
- $f_{7/2}$

$E_{vc}$

- $s_{1/2} -1.158$
- $p_{3/2} -1.587$
Breakup of $^7\text{Be}$: CDCC equations

- We have 72 coupled channels, each labeled by the set of quantum numbers
  \[ \alpha \equiv (i \ l \ j \ L) \]

- Solve set of radial coupled equations
  \[
  \left[ T_R^L + V_{\alpha\alpha'}^J (R) - E_\alpha \right] u_\alpha^J (R) = - \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'}^J (R) u_{\alpha'}^J (R) \quad (1)
  \]

- Where the coupling potential from state $\alpha$ to state $\alpha'$ is
  \[
  V_{\alpha\alpha'}^J (R) = \left\langle \left[ \phi_{\alpha'} (\bar{r}) \otimes Y_L^j (\hat{R}) \right]_{JM} \left| V(\bar{r}, \hat{R}) \left[ \phi_{\alpha} (\bar{r}) \otimes Y_L^j (\hat{R}) \right]_{JM} \right\rangle
  \]

  \[
  V(\bar{r}, \hat{R}) = U_{cT} (\bar{R}_{cT}) + U_{vT} (\bar{R}_{vT})
  \]

  and the cluster target potentials include both Coulomb and Nuclear parts

Coulomb breakup $\equiv$ neglect Nuclear in coupling potentials on rhs of Eq. (1)
Nuclear breakup $\equiv$ neglect Coulomb in coupling potentials on rhs of Eq. (1)
Breakup of $^7\text{Be}$: inputs

$^7\text{Be}$ modeled as 2-body projectile

$^3\text{He}$ spin 1/2

$^4\text{He}$ spin 0

$l=1$ relative motion

potential fitted to:
- ground and excited state energies
- 7/2- and 5/2- resonances
- scattering phase shifts
- charge radius, quadrupole and octupole moments and B(E2) for $^7\text{Li}$

wavefunction has node due to Pauli blocking

parity dependent potential

Buck et al.

cluster-target potentials from elastic scattering

$^3\text{He} + ^{208}\text{Pb} @ 300 \text{ MeV}$

$^4\text{He} + ^{208}\text{Pb} @ 400 \text{ MeV}$

$^3\text{He} + ^{12}\text{C} @ 75 \text{ MeV}$

$^4\text{He} + ^{12}\text{C} @ 100 \text{ MeV}$

NSCL

Texas A & M
Breakup of $^7$Be: convergence

$^7$Be + $^{208}$Pb $\rightarrow$ $^3$He + $^4$He + $^{208}$Pb

$E_{\text{lab}} = 100$ MeV/nucleon

Convergence checked for many variables, such as:
- Potential multipoles ($Q \leq 2$)
- Max radius for coupled equations ($R_{\text{max}} = 1000\text{fm}$)
- Maximum relative energy ($E_{\text{max}} = 20$ MeV)
- Number of partial waves in coupled equations ($L_{\text{max}} = 10000, 2000$ using interpolation)
- Bin radius and step length for coupling integrals ($r_{\text{bin}} = 50\text{fm}, h = 0.04\text{fm}$)
- Number of partial waves for breakup states ($l \leq 3$)
- Iterative solution of coupled equations versus solving set of equations exactly

[PRC 70 (2004) 011602]
Breakup of $^7$Be: higher order couplings

$^7$Be + $^{208}$Pb $\rightarrow$ $^3$He + $^4$He + $^{208}$Pb
$E_{\text{lab}} = 100$ MeV/nucleon

$^7$Be + $^{12}$C $\rightarrow$ $^3$He + $^4$He + $^{12}$C
$E_{\text{lab}} = 25$ MeV/nucleon

[PRC 70 (2004) 011602]
Breakup of $^7$Be on C: peripherality

ANC method requires that reaction is peripheral and therefore only probes the tail of the wavefunction.

Approx 28% of breakup cross section comes from impact parameters less than sum of radii.

Impact parameter from semi-classical relation $J=Kb$.

$^7$Be + $^{12}$C $\rightarrow$ $^3$He + $^4$He + $^{12}$C

$E_{lab} = 25$ MeV/nucleon

[PRC 70 (2004) 011602]
Breakup of $^7\text{Be}$ on C: Coulomb free?

Note that Coulomb is not negligible

$^7\text{Be} + ^{12}\text{C} \rightarrow ^3\text{He} + ^4\text{He} + ^{12}\text{C}$

$E_{\text{lab}} = 25$ MeV/nucleon

[PRC 70 (2004) 011602]
Breakup of $^7$Be: nuclear free?

$^7$Be + $^{208}$Pb $\rightarrow$ $^3$He + $^4$He + $^{208}$Pb

$E_{\text{lab}}$ = 100 MeV/nucleon

Impact parameter from semi-classical relation $J = Kb$

Coulomb dissociation method requires that the reaction is Coulomb dominated, outside the range of the nuclear force.

Maximum range of nuclear force assuming Rutherford trajectories.

This impact parameter relates to a scattering angle of 2.5°.

[PRC 70 (2004) 011602]
Breakup of $^7\text{Be}$: nuclear component

$^7\text{Be} + ^{208}\text{Pb} \rightarrow ^3\text{He} + ^4\text{He} + ^{208}\text{Pb}$

$E_{\text{lab}} = 100$ MeV/nucleon

Cross section dominated by nuclear breakup even for small scattering angles below the cut-off angle

Cut-off angle from Rutherford orbit = 2.5°
Breakup of $^7$Be: small relative energies

Angular cross section summed for only first 2 bins in each $j^\pi$ set

This dramatically reduces the amount of nuclear breakup in the forward angles

$^7$Be + $^{208}$Pb → $^3$He + $^4$He + $^{208}$Pb

$E_{\text{lab}} = 100$ MeV/nucleon

restricted maximum relative energy between $^3$He and $^4$He fragments in final state

$E_{\text{vc}}$ All bins included in coupled channels solution

$\theta_{\text{c.m.}}$ (deg)

$E_{\text{rel}} < 1$ MeV

$^7$Be + $^{208}$Pb → $^3$He + $^4$He + $^{208}$Pb

$E_{\text{lab}} = 100$ MeV/nucleon

restricted maximum relative energy between $^3$He and $^4$He fragments in final state

$E_{\text{vc}}$ All bins included in coupled channels solution

Angular cross section summed for only first 2 bins in each $j^\pi$ set

This dramatically reduces the amount of nuclear breakup in the forward angles

$p_{1/2} \quad -1.158$

$p_{3/2} \quad -1.587$

$p_{1/2} \quad -1.158$

$p_{3/2} \quad -1.587$

$s_{1/2} \quad p_{1/2} \quad p_{3/2} \quad d_{3/2} \quad d_{5/2} \quad f_{5/2} \quad f_{7/2}$

[PRC 70 (2004) 011602]
Breakup of $^7$Be: E1 versus E2

$^7$Be + $^{208}$Pb → $^3$He + $^4$He + $^{208}$Pb
$E_{\text{lab}} = 100$ MeV/nucleon

restricted maximum relative energy between $^3$He and $^4$He fragments in final state
Breakup of $^7$Be: cont-cont couplings

Restricted maximum relative energy between $^3$He and $^4$He fragments in final state

DWBA $\rightarrow$ first order

No CC couplings includes couplings to all orders to/from ground state only

[PRC 70 (2004) 011602]
Breakup of $^7$Be: comparison with other models

$^7$Be + $^{208}$Pb $\rightarrow$ $^3$He + $^4$He + $^{208}$Pb

$E_{\text{lab}} = 100$ MeV/nucleon

Semiclassical A&W
Coulomb-1step
Coulomb-CDCC

[PRC 70 (2004) 011602]
CDCC cross sections

1) Calculate:
   • projectile wfn's: bound and scattering states
   • bins wfn's
   • coupling potentials
2) Solve coupled channel equations and match to obtain S-matrix
3) Construct observables.

Scattering amplitude

\[ \tilde{F}_{M'M'}(K_{p'}) = \frac{4\pi}{K_0} \sqrt{\frac{K_{p'}}{K_0}} \sum_{L L' J} \langle L0, I_p M | J M \rangle \langle L' M - M', I_{p'} M' | J M \rangle \]
\[ \times \exp(i[\sigma_L + \sigma_{L'}]) \frac{1}{2i} S^{I}_{\alpha' \alpha}(p') Y_{L}^{0}(\hat{K}_0) Y_{L'}^{M - M'}(\hat{K}_{p'}). \]

\[ \frac{d\sigma(p')}{d\Omega_K} = \frac{1}{2I_p + 1} \sum_{M M'} \left| \tilde{F}_{M'M'}(K_{p'}) \right|^2 \]
Three-body observables: T-matrix

The T matrix can be constructed from the CDCC solution

\[ T_{\mu\sigma:M}(k, K) = \langle \phi_{k\mu\sigma}^{(-)}(r) e^{iK \cdot R} | U(r, R) | \Psi_{K0M}^{CDCC}(r, R) \rangle \]

\[ T_{\mu\sigma:M}(k, K) = \sum_{p,M'} \langle \phi_{k\mu\sigma}^{(-)} | \tilde{\phi}_{p}^{M'} \rangle \langle \tilde{\phi}_{p}^{M'} e^{iK \cdot R} | U(r, R) | \Psi_{M}(r, R) \rangle \]

\[ \langle \tilde{\phi}_{k\mu\sigma}^{(-)} | \phi_{p}^{M'} \rangle = \frac{(2\pi)^{3/2}}{k \sqrt{N_{\alpha}}} \sum_{v} (-i)^{\ell} \langle \ell v, s\sigma | jm, I_{c\mu} | I_{p}^{M'} \rangle e^{i\delta_{p}(k)} \tilde{u}_{p}(k) Y_{\ell v}(k) \]

\[ \hat{T}_{M'M}^{p}(K_{p}) = \langle \hat{\phi}_{p}^{M'} e^{iK_{p} \cdot R} | U(r, R) | \Psi_{K0M}^{CDCC}(r, R) \rangle = -\frac{2\pi \hbar^{2}}{\mu_{(vc)t}} \sqrt{\frac{K_{0}}{K_{p}}} \hat{F}_{M'M}^{p}(K_{p}) \]
Let's say we measure the energy of one fragment $c$ and the angles of both:

\[
\frac{d^3 \sigma}{d \Omega_c d \Omega_v d E_c} = \frac{2\pi \mu_{(vc) t}}{\hbar^2 K_0} \frac{1}{(2I_p + 1)} \sum_{\mu \sigma M} \left| T_{\mu \sigma : M} (k, K) \right|^2 \rho_{ps} (E_c, \Omega_c, \Omega_v),
\]

Phase space factor

\[
\rho_{ps} (E_c, \Omega_c, \Omega_v) dE_c d\Omega_c d\Omega_v =
\]

\[
\int d^3 k_c d^3 k_v d^3 k_t \delta (P - \hbar k_c - \hbar k_v - \hbar k_t) \delta (E - E_c - E_v - E_t),
\]
CDCC three-body observables

Let’s say we only measure fragment c both energy and angles, with a certain solid angle and an efficiency dependent on angle:

\[
\left\langle \frac{d^2\sigma}{d\Omega_c dE_c} \right\rangle = \frac{1}{\Delta \Omega_c} \int_{\Delta \Omega_c} d\Omega_c \left\{ e(\Omega_c) \int d\Omega_v \frac{d^3\sigma}{d\Omega_c d\Omega_v dE_c} \right\}
\]

Other typical observables: momentum distributions

\[\text{FIG. 4: Momentum distribution of } ^7\text{Be following the breakup of 44 MeV/A } ^8\text{B on a } ^{208}\text{Pb target integrated up to } \theta_{\text{max}} = 1.5^\circ: \text{ the previous results [8] with no renormalisation of the quadrupole excitation (solid line) and with a 1.6 renormalization (dot-dashed line) and the result using 0.5*V(p-Be) for the final state (dashed line).}\]