

# Perspectives and limitations of 2p and 2n knockout reactions

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# Acknowledge the ongoing (growing) collaboration

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# Towards 2N spectroscopy of rare isotope beams

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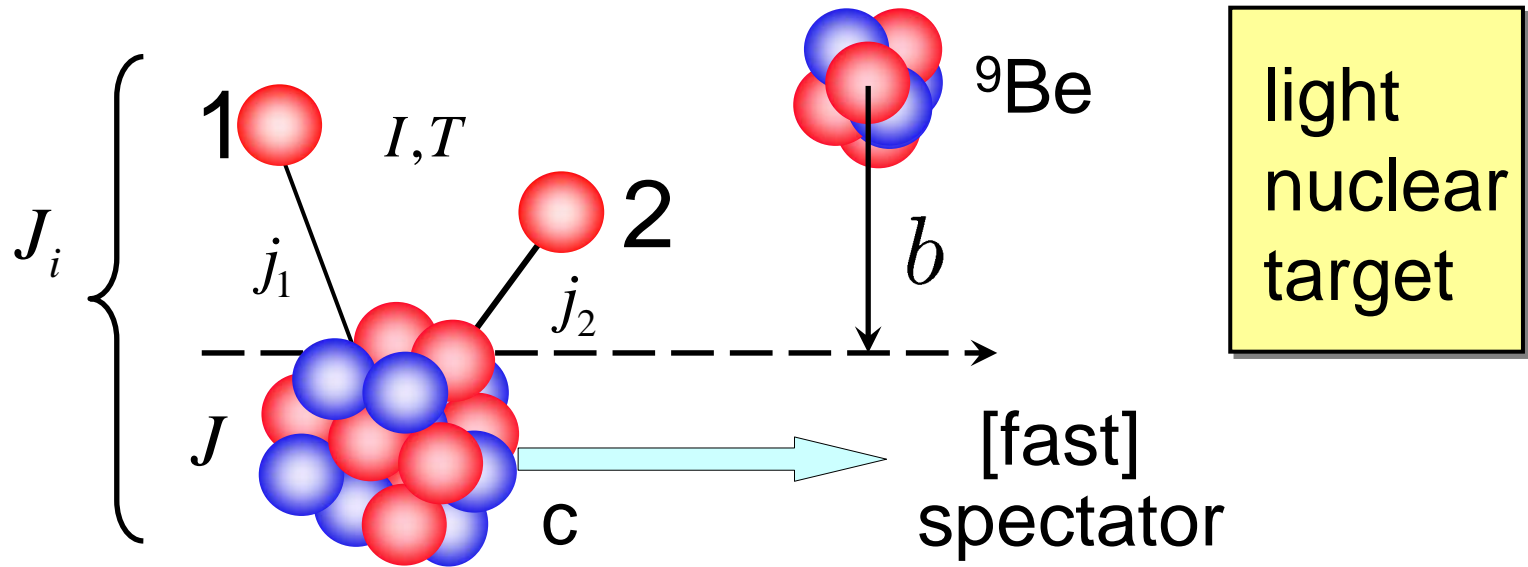
## Outline

Knockout mechanism: (i) Sudden two nucleon (2N) removal from fast fragmentation beams, (ii) the 'parameter selection' scheme – where input is taken/guided by theory, (iii) recent results and status check, (iv) limitations – but still early days.

Interest: (i) assessing shell model wave functions and interactions, (ii) shell gaps & 2N correlations, as revealed by inclusive and partial cross sections, -2N momentum distributions



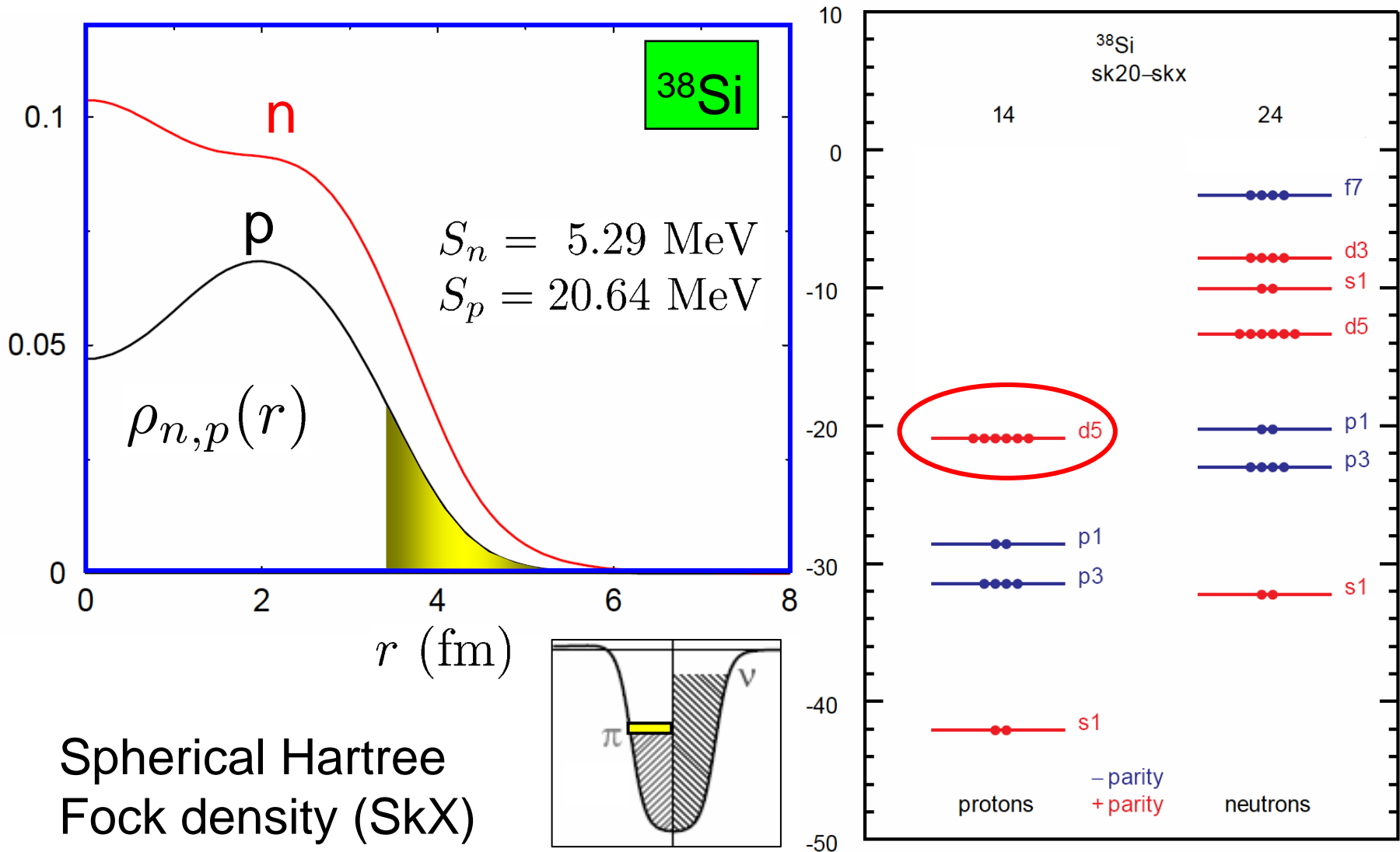
## 2N removal at beam energies $> 100$ MeV/nucleon



Experiments are inclusive (with respect to the target final states). Residue final state measured – using gamma rays, whenever possible – and momenta ( $p_{//}$ ) of the residues.

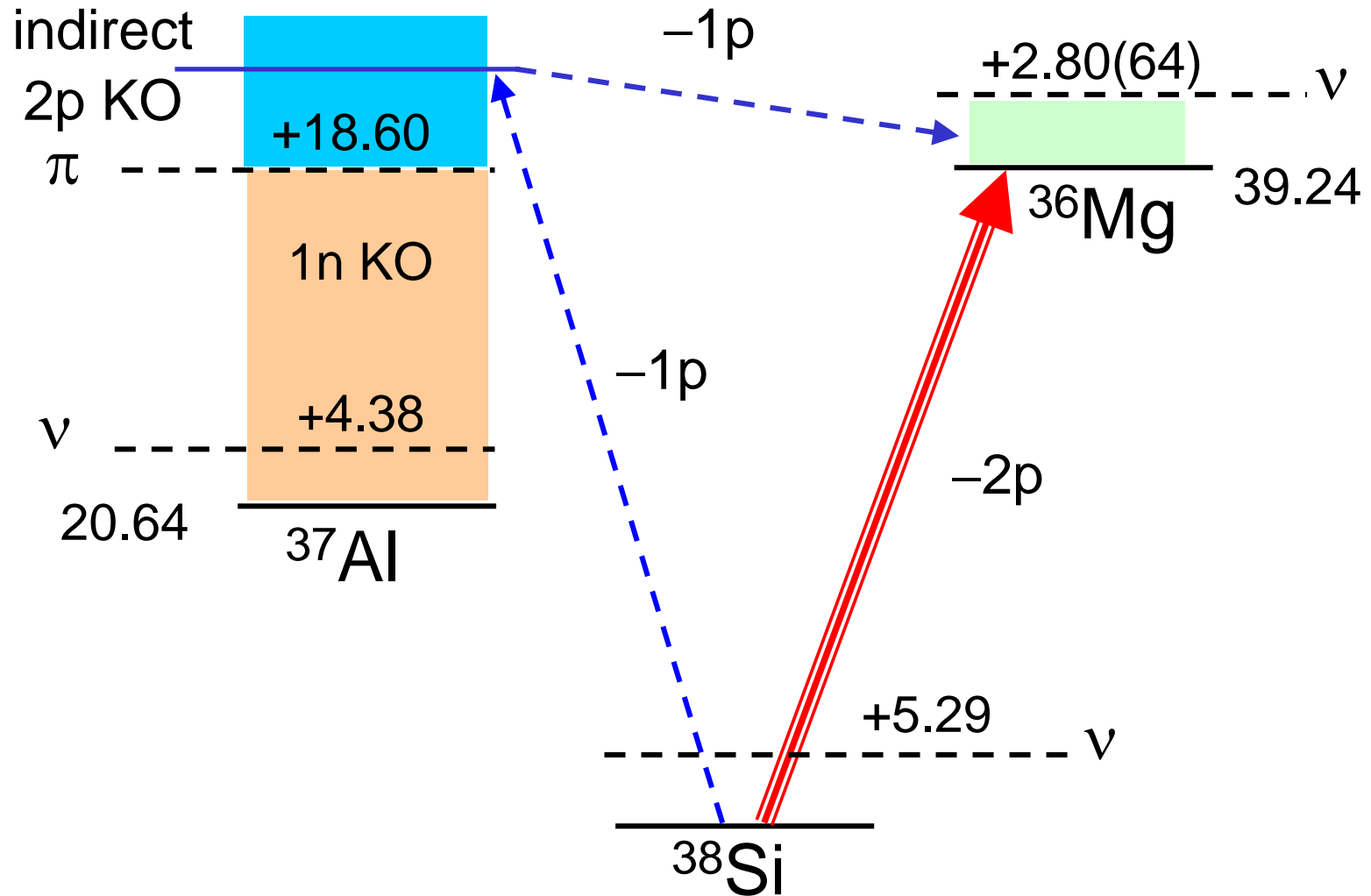
Cross sections are large and they include both:  
Stripping (inelastic/absorptive) and diffractive (elastic) interactions of the removed nucleon(s) with the target

# 2N removal – removal of deficient species - direct

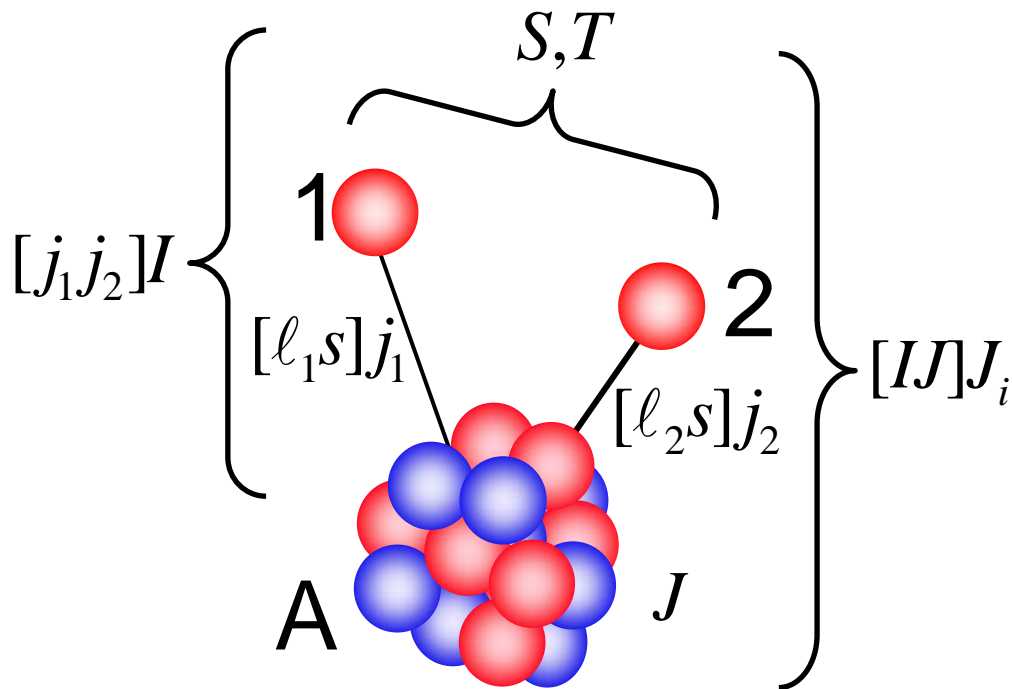


Spherical Hartree  
Fock density (SkX)

# Energetics: 2p knockout: $^{38}\text{Si} \rightarrow ^{36}\text{Mg}$



# Sudden removal from the residue as a spectator

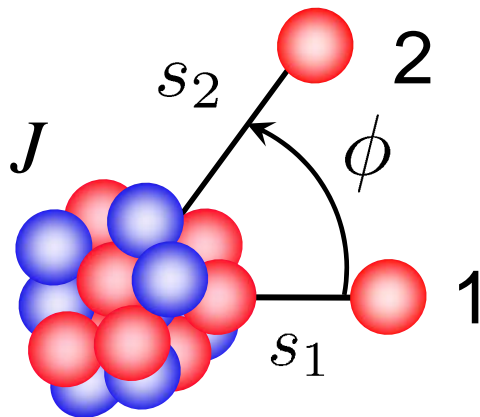
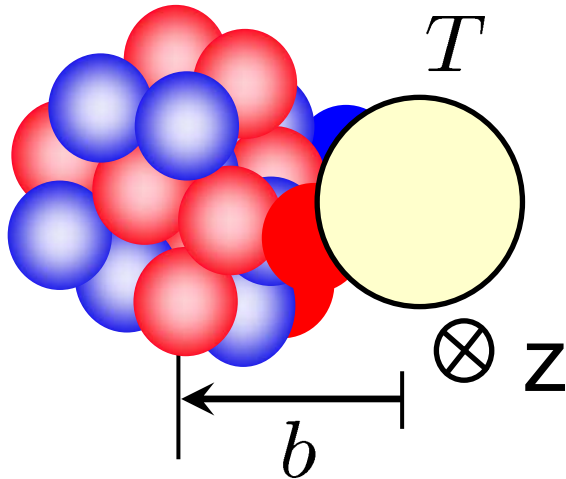


Core/residue state is assumed a spectator – so reaction probes the two nucleon overlap and (in general) there are several coherent active 2N configurations – overlap determined by the two nucleon amplitudes (TNA)

$$F_{JM}(1, 2) = \langle 1, 2, \Phi_{c, JM}(A) | \Phi_{A+2} \rangle \text{ and with } J_i = 0^+$$

$$F_{JM}(1, 2) = \sum_{j_1 j_2} (-)^{J+M} C(j_1 j_2 J) / \hat{J} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{J-M}$$

# Target drills out a cylindrical volume at the surface



(i) Cross section will be sensitive to the spatial correlations of pairs of nucleons near surface

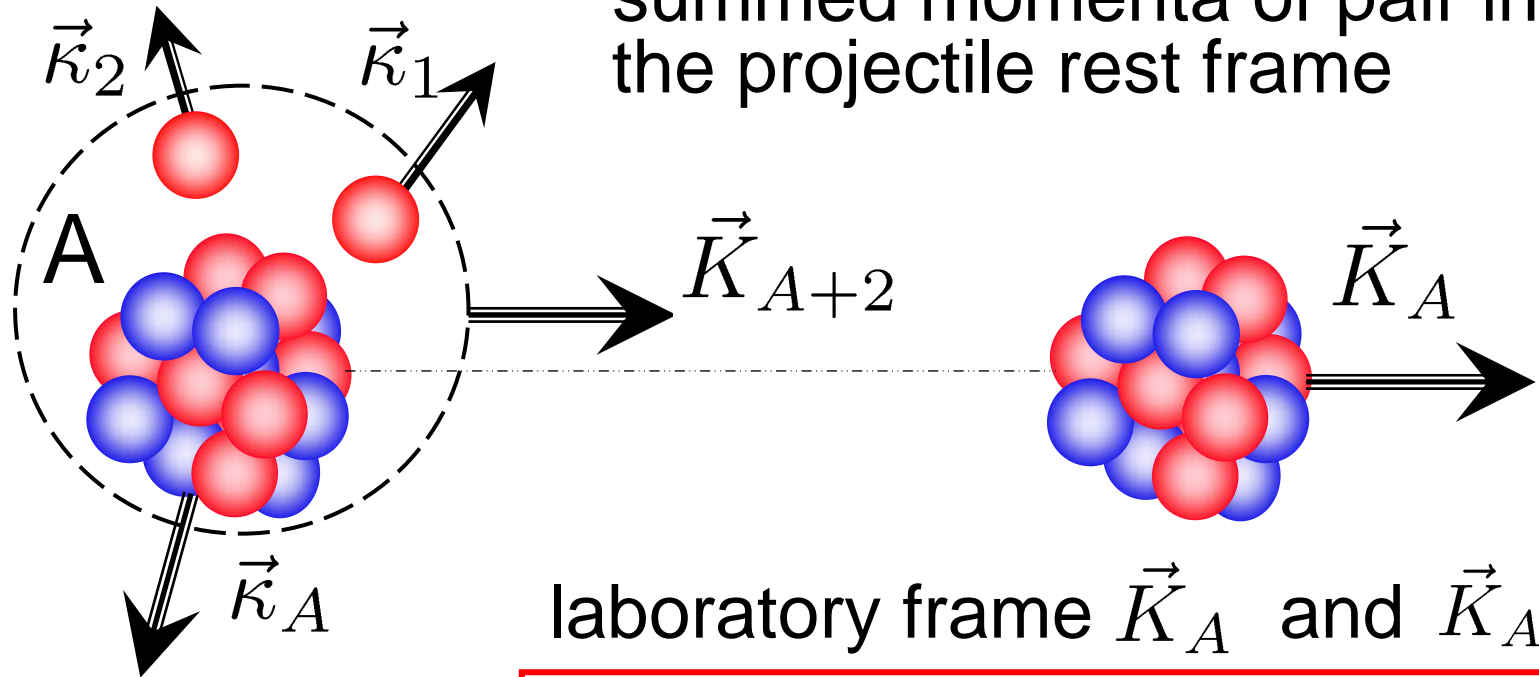
(ii) No spin selection rule (for  $S=0$  versus  $S=1$  pairs). Reaction mechanism

(iii) We can gain first expectation of the extent to which we are sensitive to correlations by looking at the  $2N$  overlaps in this sampled volume

$$P_J(\vec{s}_1, \vec{s}_2) = \sum_M \int dz_1 \int dz_2 \langle F_{JM}(1, 2) | F_{JM}(1, 2) \rangle_{sp}$$

# Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame



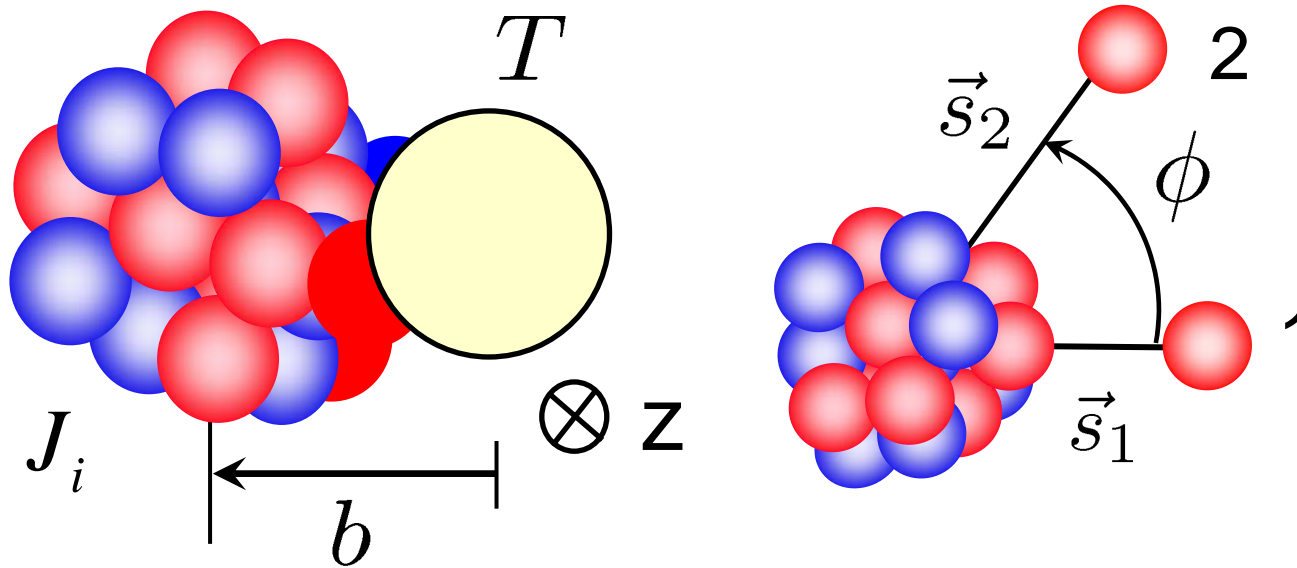
Projectile rest frame

laboratory frame  $\vec{K}_A$  and  $\vec{K}_{A+2}$

$$\vec{K}_A = \frac{A}{A+2} \vec{K}_{A+2} - [\vec{k}_1 + \vec{k}_2]$$

and component equations

# Look at momentum content of sampled volume

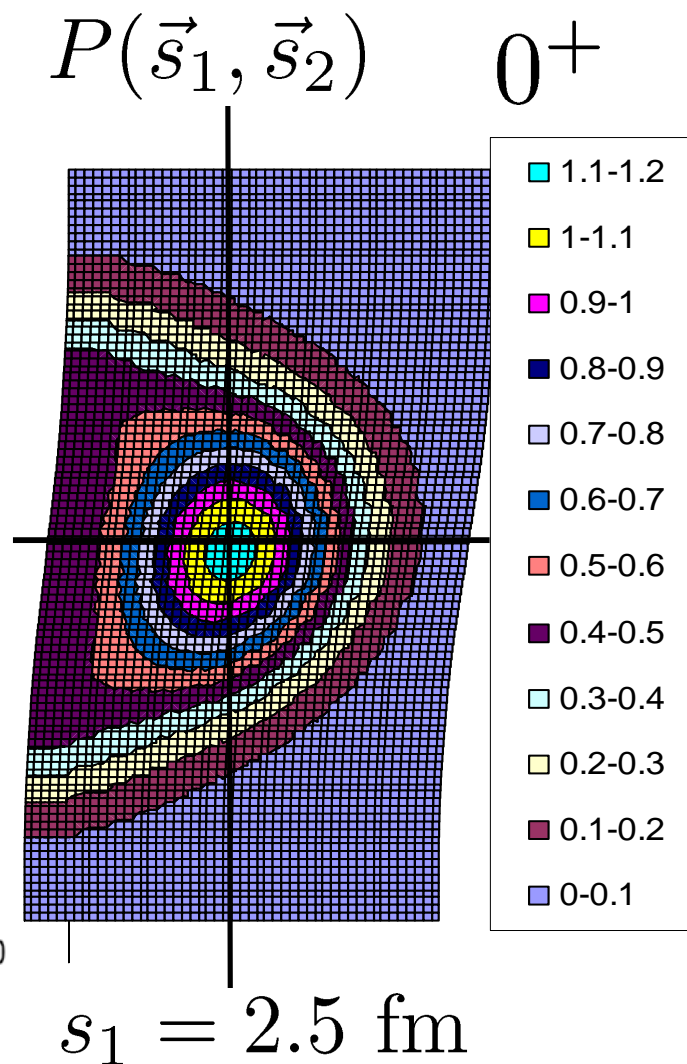
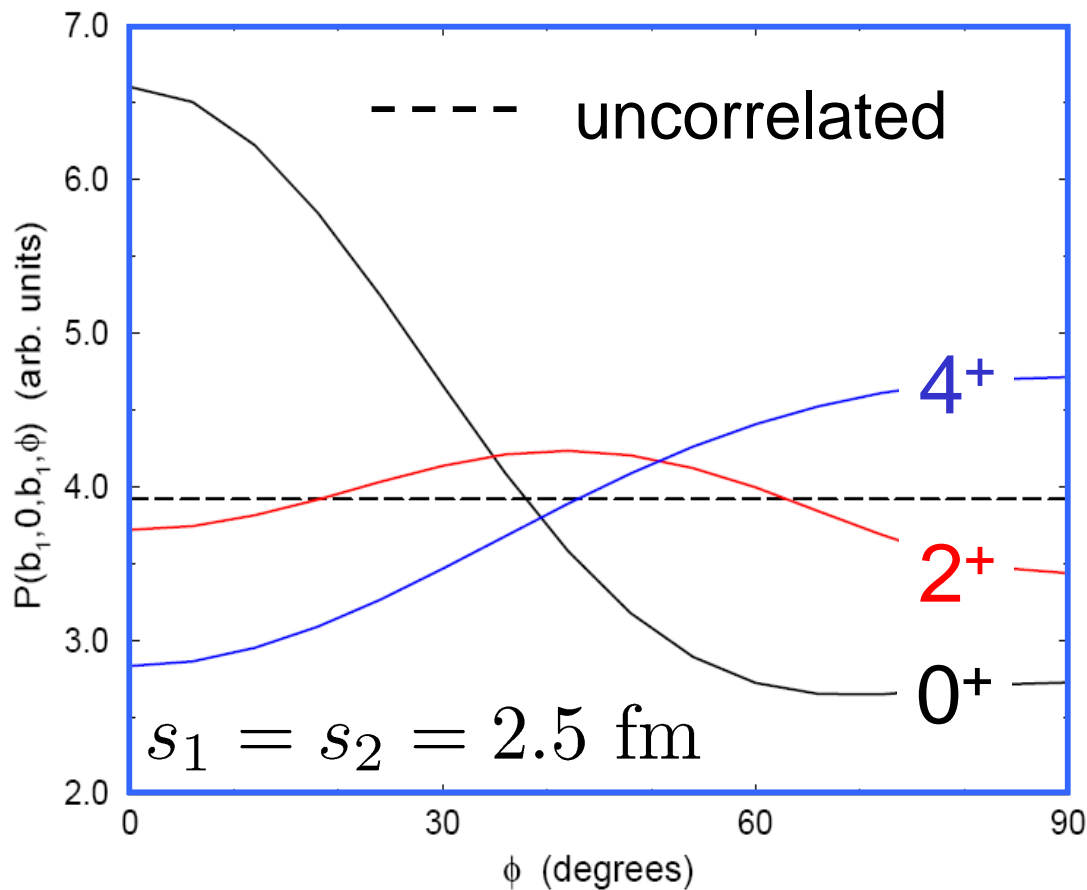


Probability of a residue with parallel momentum  $\kappa_A$

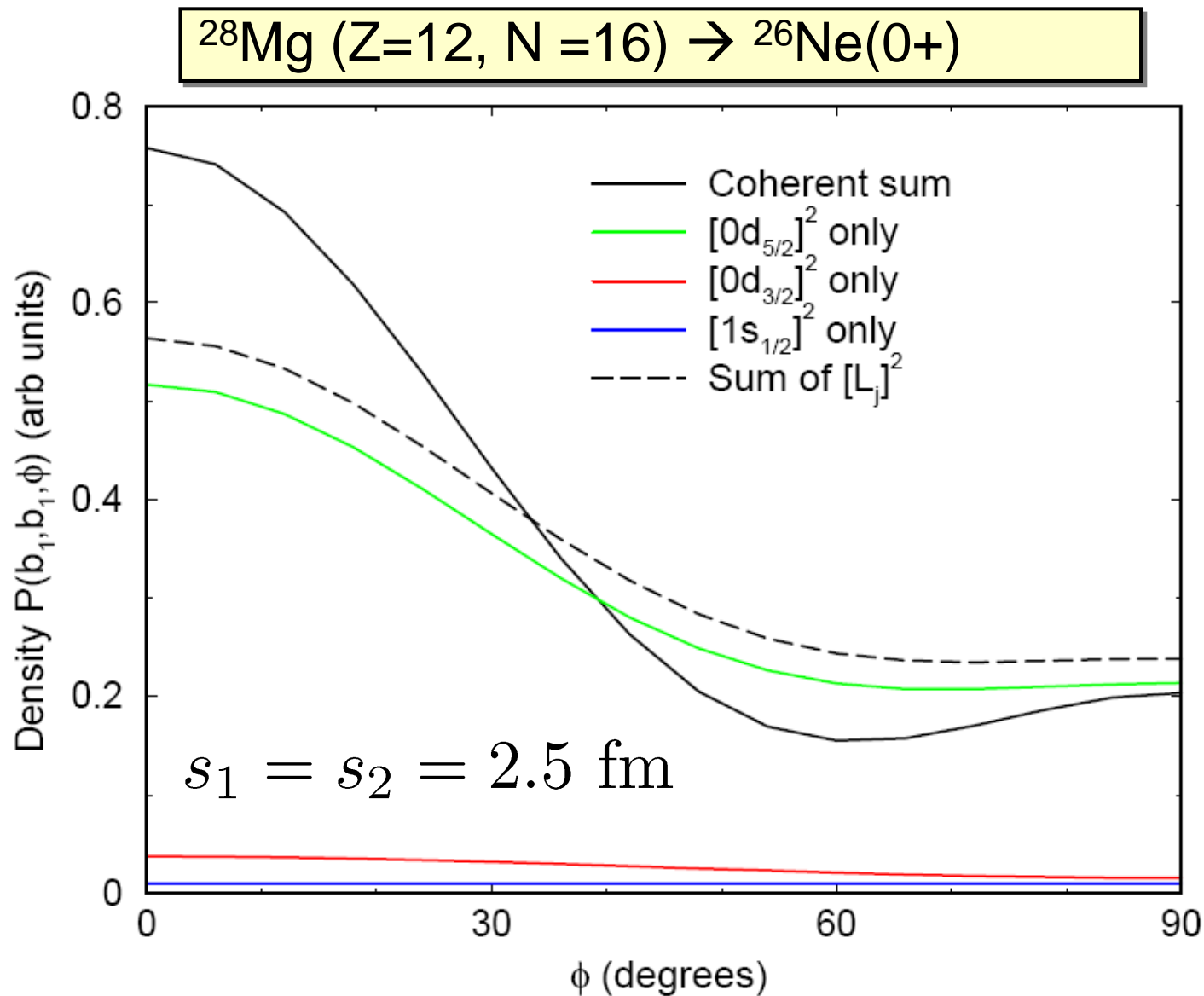
$$P(\kappa_A, \vec{s}_1, \vec{s}_2) = \sum_M \left\langle \int d\kappa_1 \int d\kappa_2 \delta(\kappa_A + \kappa_1 + \kappa_2) \right. \\ \left. \times \left| \int dz_1 \int dz_2 e^{i\kappa_1 z_1} e^{i\kappa_2 z_2} F_{JM}(1, 2) \right|^2 \right\rangle_{sp}$$

# Antisymmetrized $^{28}\text{Mg} \rightarrow ^{26}\text{Ne}$ removal of $\pi[1d_{5/2}]^2$

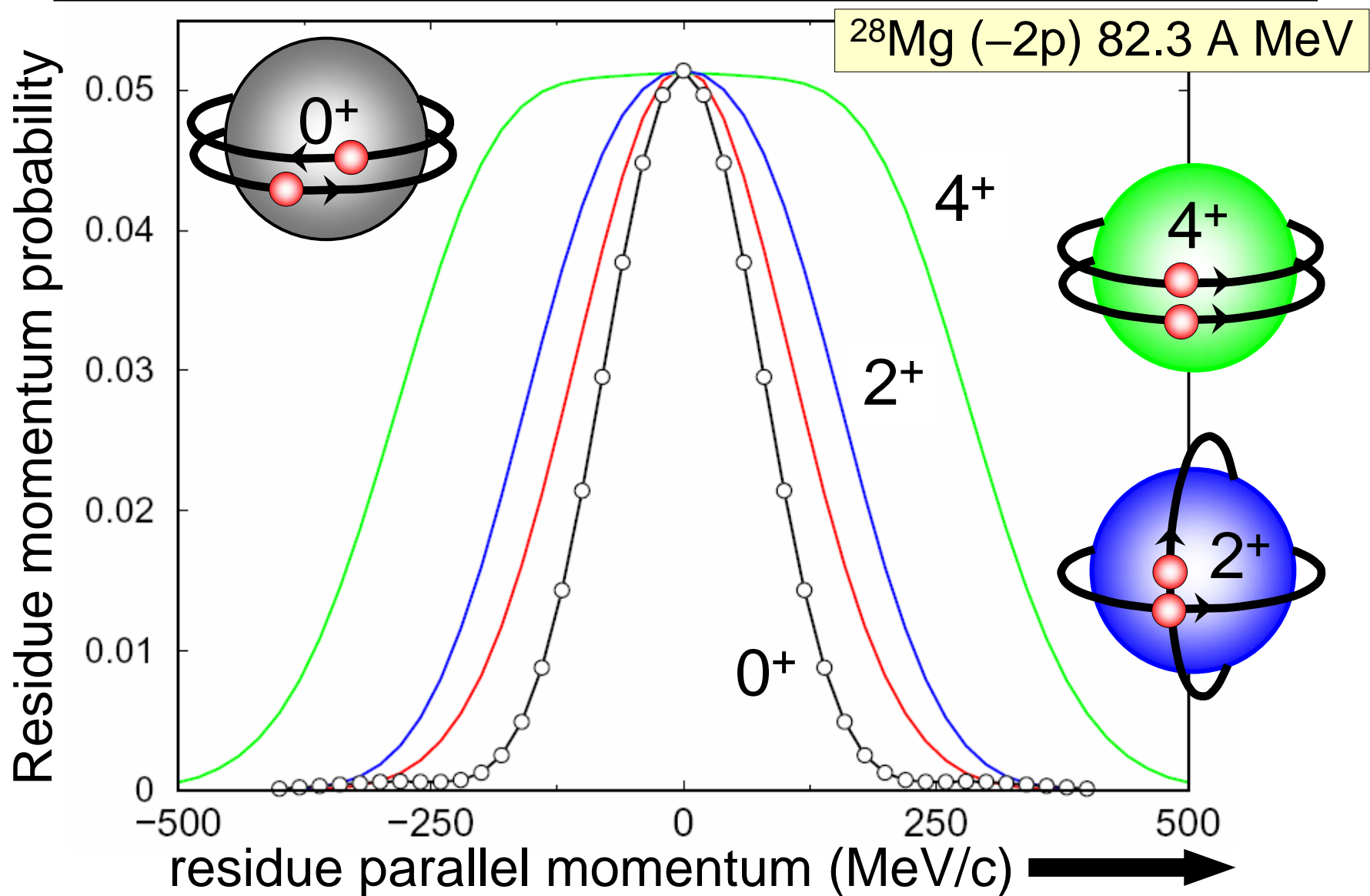
$$F_{JM} \propto [\overline{\phi_j \otimes \phi_j}]_{J-M}$$



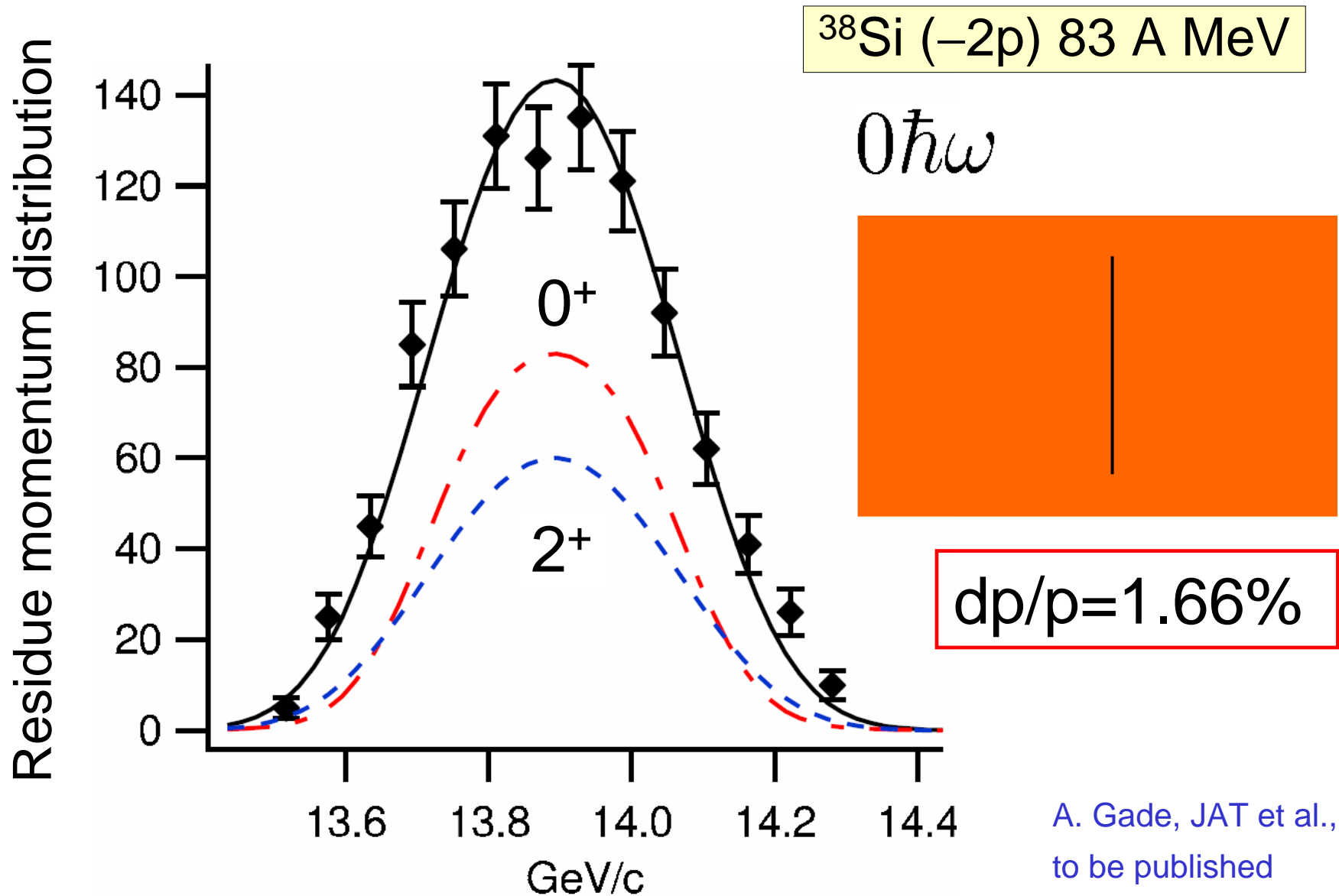
# Coherence of shell model correlations



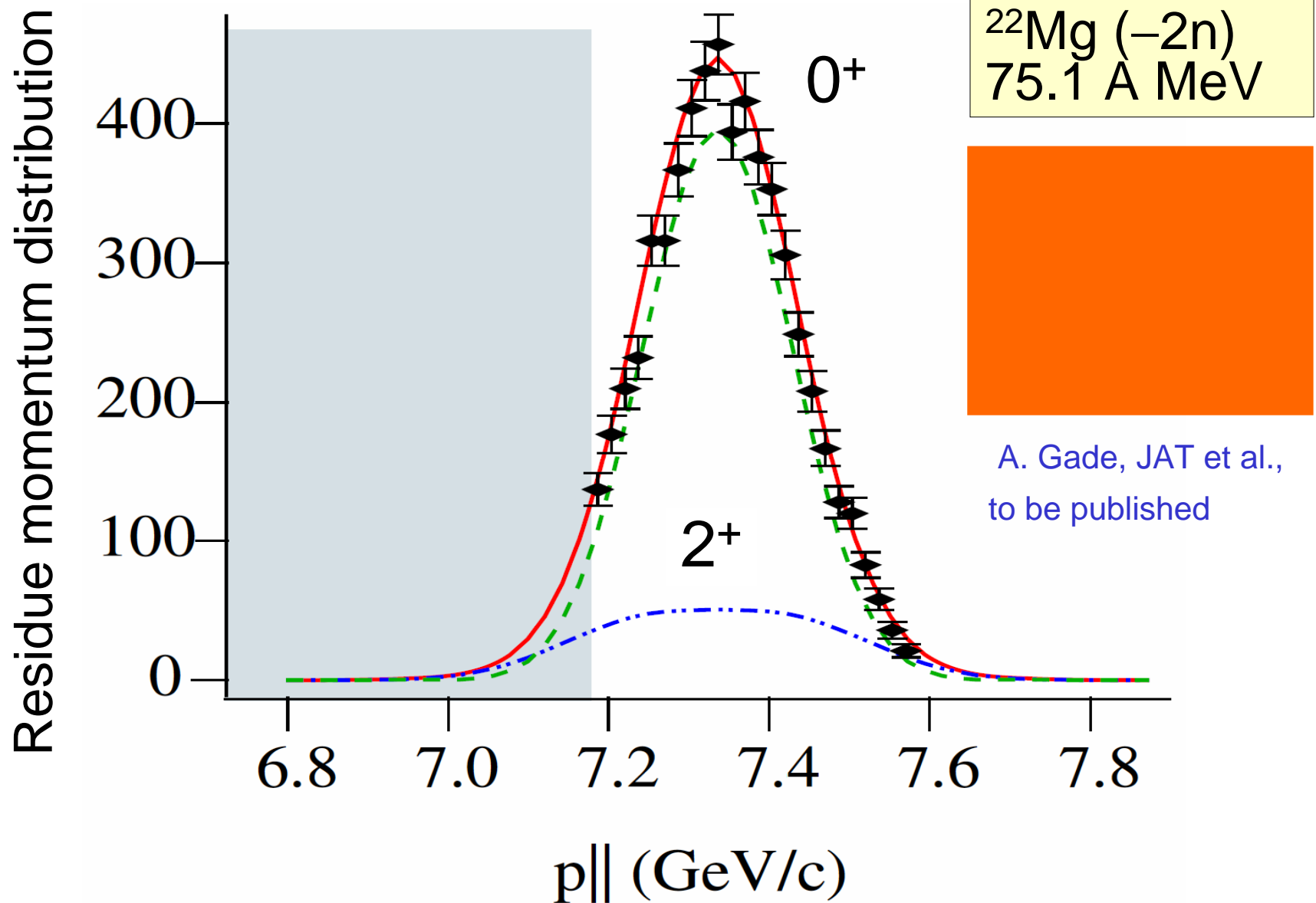
# Two nucleon KO –J-dependence of $p_{\parallel}$



# Two proton knockout from $^{38}\text{Si} \rightarrow ^{36}\text{Mg}(0^+, 2^+)$



# Two neutron knockout from $^{22}\text{Mg} \rightarrow ^{20}\text{Mg}(0^+, 2^+)$



## Now include all 2N absorption mechanisms

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$$\sigma_{abs} \rightarrow 1 - |S_c|^2 |S_1|^2 |S_2|^2$$

$$\begin{aligned}
 1 &= \left[ |S_c|^2 + \cancel{(1 - |S_c|^2)} \right] \\
 &\times \left[ |S_1|^2 + (1 - |S_1|^2) \right] \\
 &\times \left[ |S_2|^2 + (1 - |S_2|^2) \right]
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1 \\ \times \\ \times \end{aligned}} \right\} \begin{array}{l} \text{core survival} \\ \text{and nucleon} \\ \text{“removal”} \end{array}$$

$$\begin{aligned}
 \sigma_{abs}^{KO} &\rightarrow |S_c|^2 (1 - |S_1|^2)(1 - |S_2|^2) && \text{2N stripping} \\
 &+ |S_c|^2 |S_1|^2 (1 - |S_2|^2) && \left. \vphantom{\begin{aligned} \sigma_{abs}^{KO} \\ + \\ + \end{aligned}} \right\} \text{1N stripped} \\
 &+ |S_c|^2 (1 - |S_1|^2) |S_2|^2 && \left. \vphantom{\begin{aligned} \sigma_{abs}^{KO} \\ + \\ + \end{aligned}} \right\} \text{1N diffracted}
 \end{aligned}$$

+ 2N diffraction contributions  $\approx 6 - 8\%$

# The diffractive/stripping contributions

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$$\sigma_2 \rightarrow |S_c|^2 |S_1|^2 \underbrace{(1 - |S_2|^2)}$$

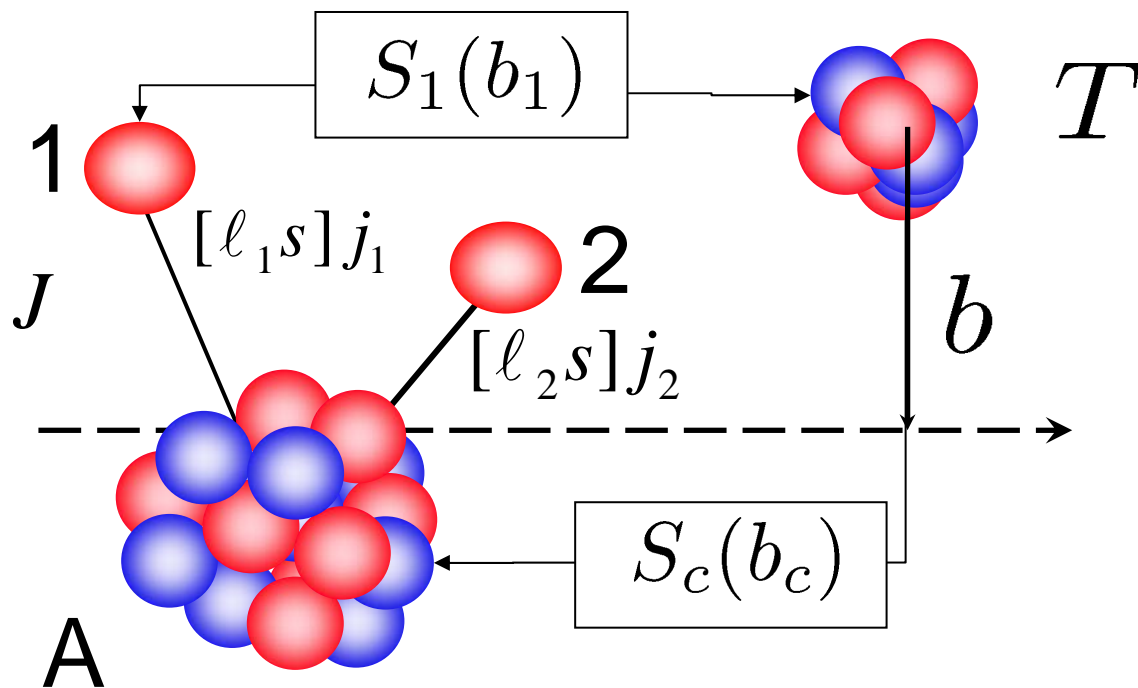
nucleon 2 absorbed

nucleon 1 survives, but can  
be ~~bound~~ to c or unbound ✓

$$|S_1|^2 = S_1^* \left[ \underbrace{\left( 1 - \sum_{\text{bound}} |\alpha\rangle\langle\alpha| \right)}_{(1+c) \text{ unbound}} + \underbrace{\sum_{\text{bound}} |\alpha\rangle\langle\alpha|}_{(1+c) \text{ bound}} \right] S_1$$

nucleon 1: (1+c) unbound (1+c) bound

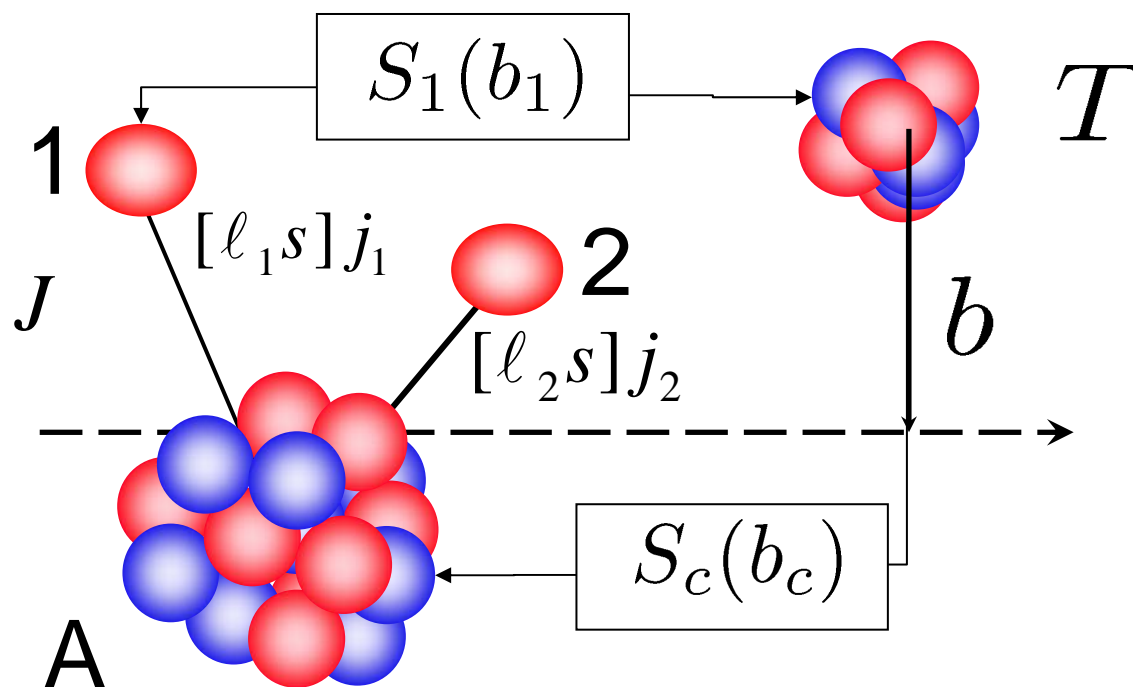
# Sudden removal – eikonal model cross sections



$$\sigma = \frac{1}{2J+1} \sum_M \int d\vec{b} \langle F_{JM} | \hat{O}(c, 1, 2) | F_{JM} \rangle$$

$$2N \text{ Stripping} : \hat{O}(c, 1, 2) = |S_c|^2 (1 - |S_1|^2) (1 - |S_2|^2)$$

# Sudden removal – eikonal model cross sections

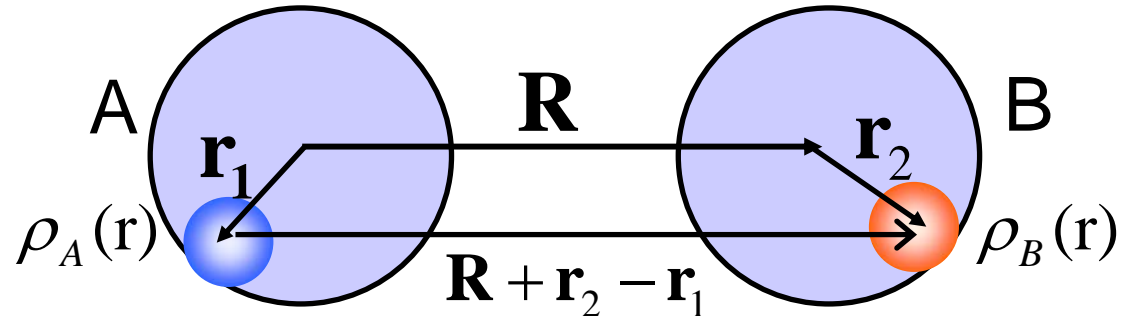


At any given facility, and a programme of measurements (with essentially fixed energy per nucleon) and a given target then only two things change for different exotic beams (1) the core target interaction, (2) the nuclear structure \*\*\*

# Effective interactions – Folding models

Double  
folding

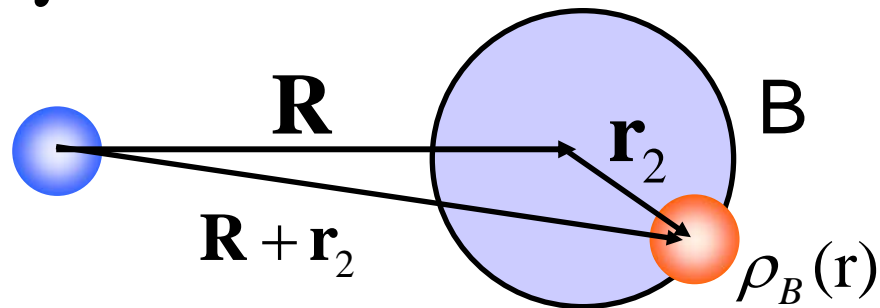
$$V_{AB}(\mathbf{R}) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_A(\mathbf{r}_1) \rho_B(\mathbf{r}_2) t_{\text{NN}}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$



$V_{cT}$

Single  
folding

$$V_B(\mathbf{R}) = \int d\mathbf{r}_2 \rho_B(\mathbf{r}_2) t_{\text{NN}}(\mathbf{R} + \mathbf{r}_2)$$



$V_{vT}$

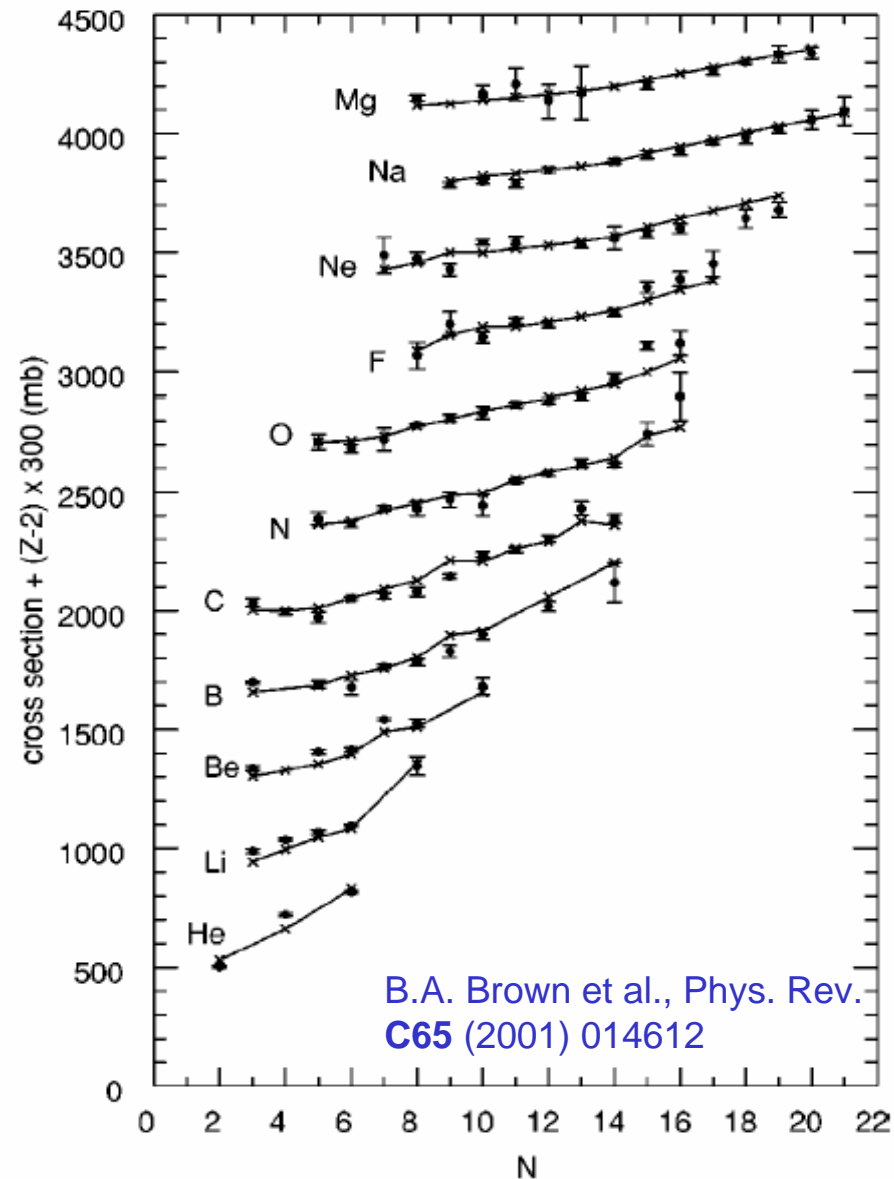
Glauber limit:  $t_{np} = \sigma_{np}[i + \alpha_{np}]f_{np}(r)/2$ , etc.

## Reaction description is rather robust - quantitative

Residue-target interaction is highly absorptive at  $>100$  MeV/u with a range 'fixed' by the residue and target sizes. This is encoded within the double folding model and is cross referenced to

$$\sigma_R = 2\pi \int db [1 - |S_c(b)|^2]$$

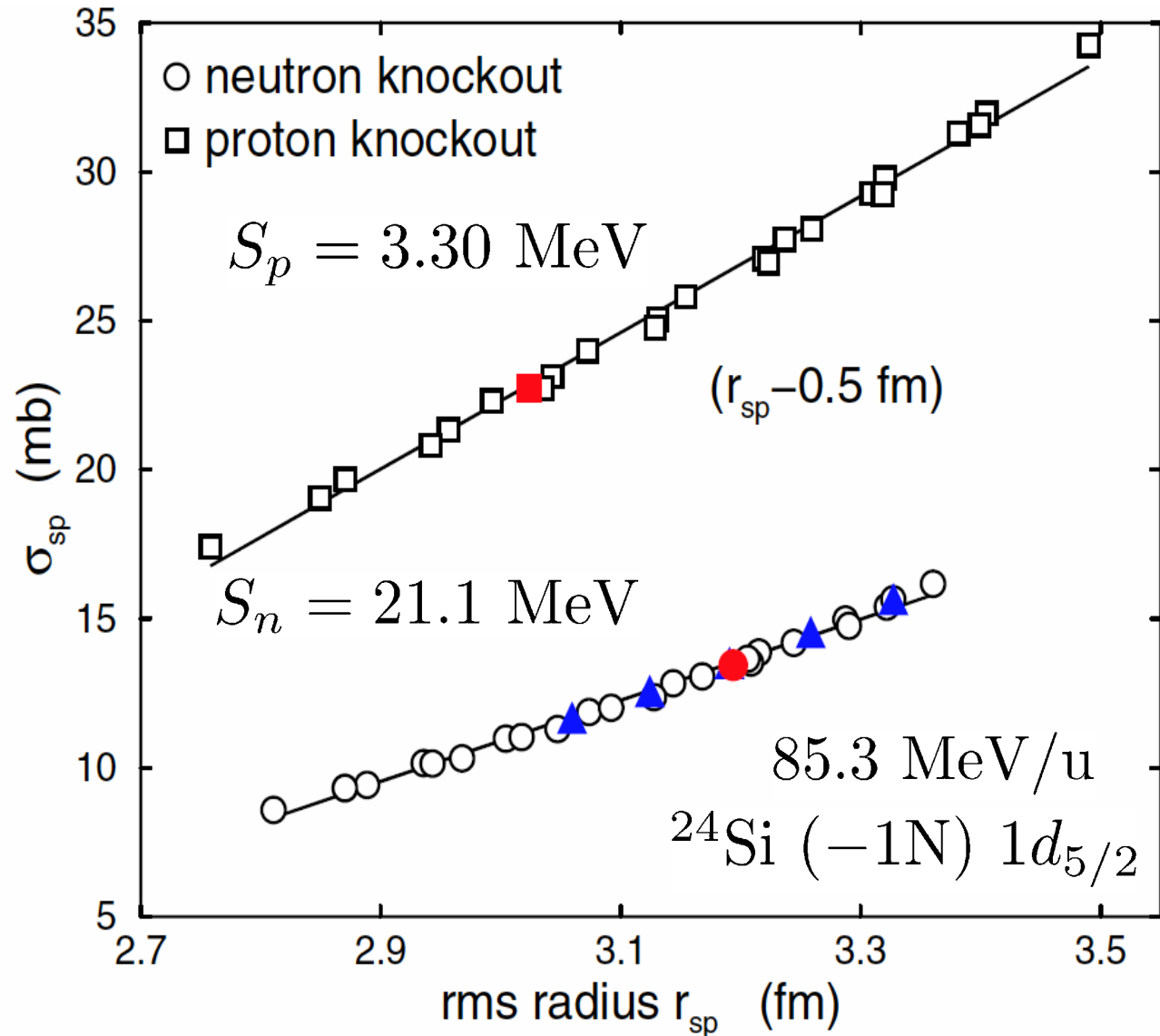
These ion-ion interactions and their S-matrices are calculated reliably using Glauber methods – using Hartree-Fock densities.



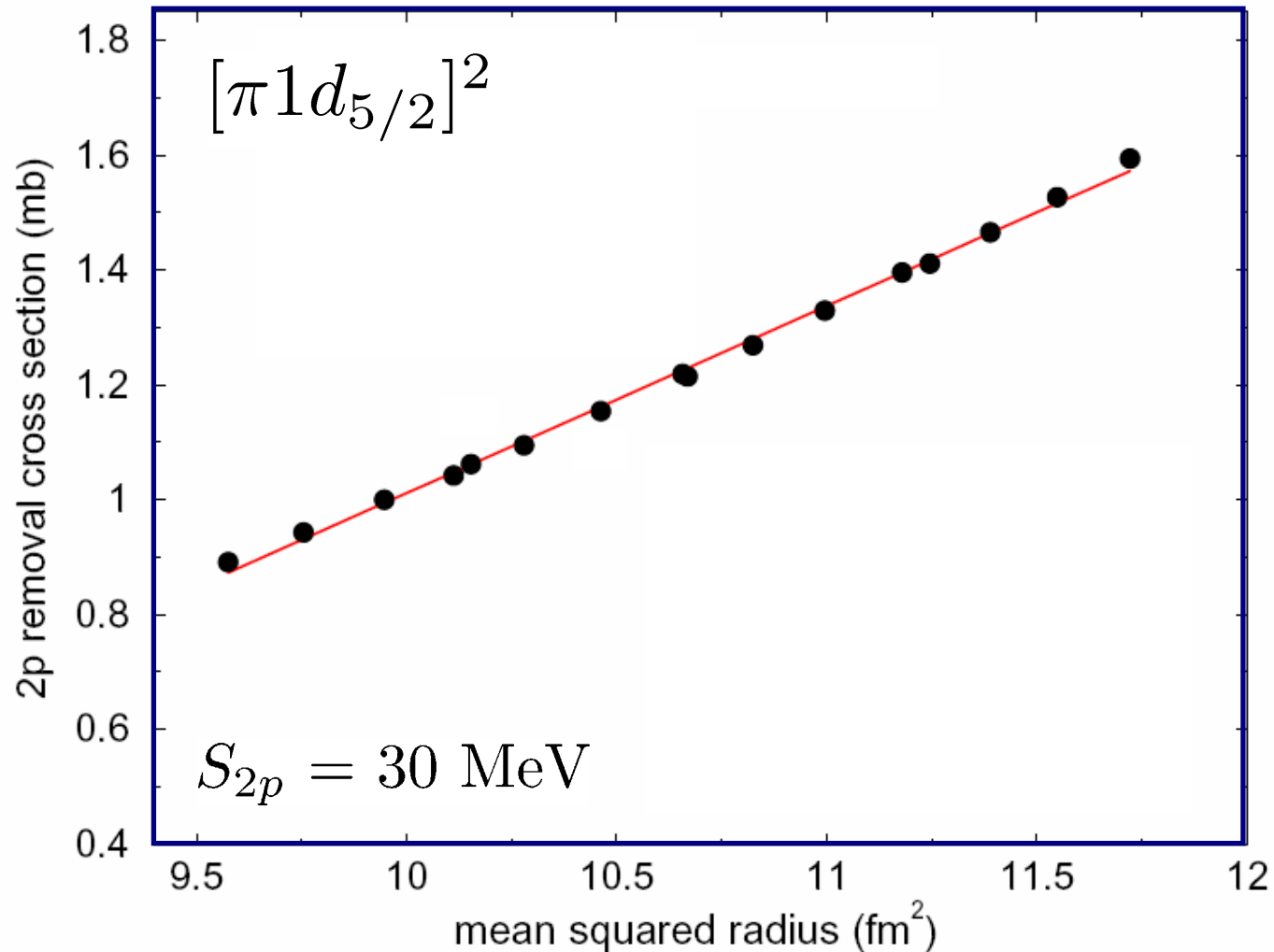
# Overlap function sensitivity: Hartree Fock 'sizes'

Rms radii of sp  
formfactors are  
the requirement  
for determining  
the cross sections  
– to high precision  
We constrain to  
Hartree-Fock  
values

$$\sigma(r_0, a_0, V_{so}, \beta_{NL})$$



# Sensitivity to s.p. orbitals – correlation with radii



## Single particle radii – other benchmarks

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Can compare sp  
rms radii in selected  
cases in the mass  
region under study  
with those used /  
deduced in electron  
scattering:  
e.g. \*\* Kramer et al.  
NPA **679** 267 (2001)

$^{16}\text{O}$

\* 2.954      2.903 fm (HF)

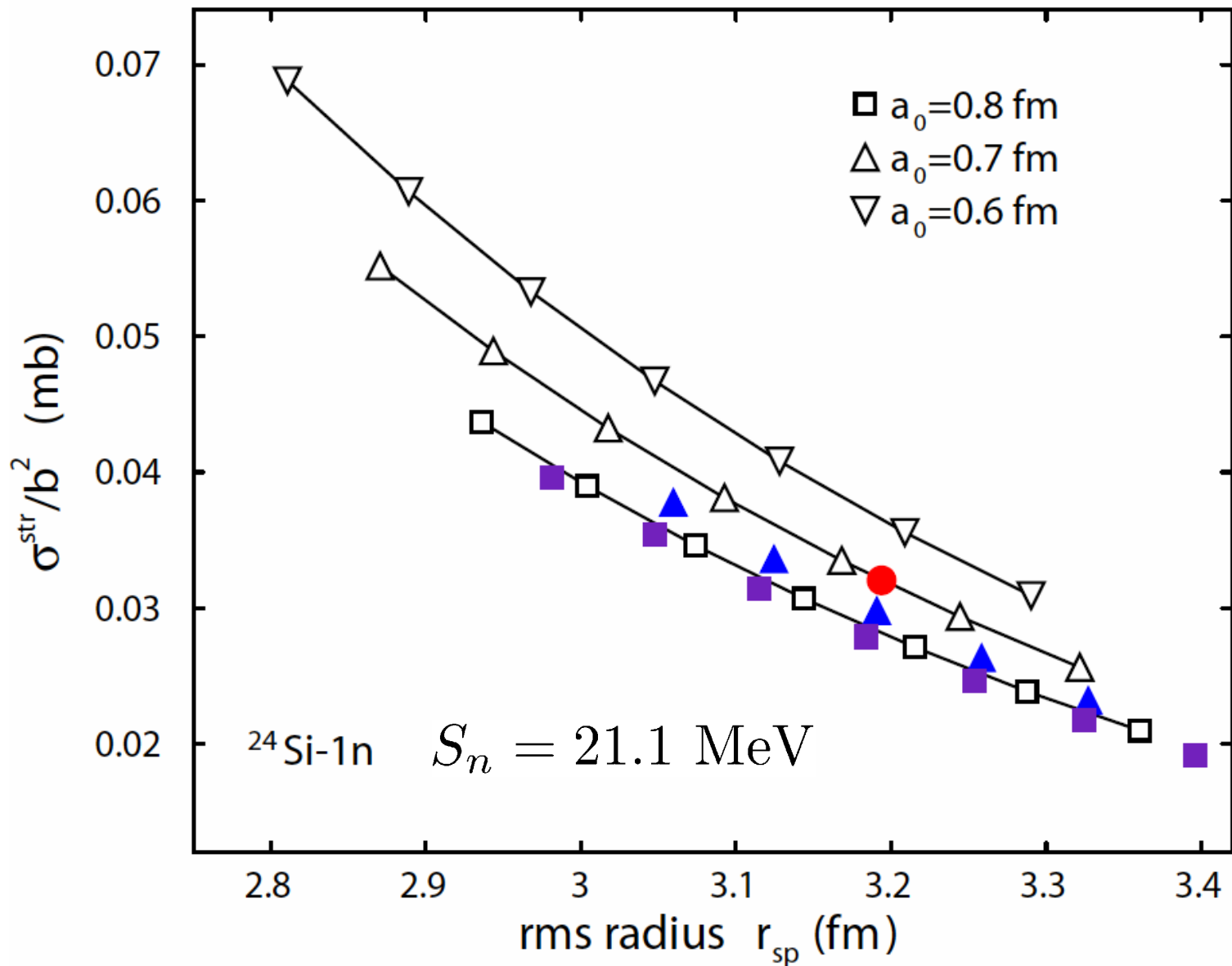
$^{40}\text{Ca}$

\* 3.712      3.670 fm (HF)

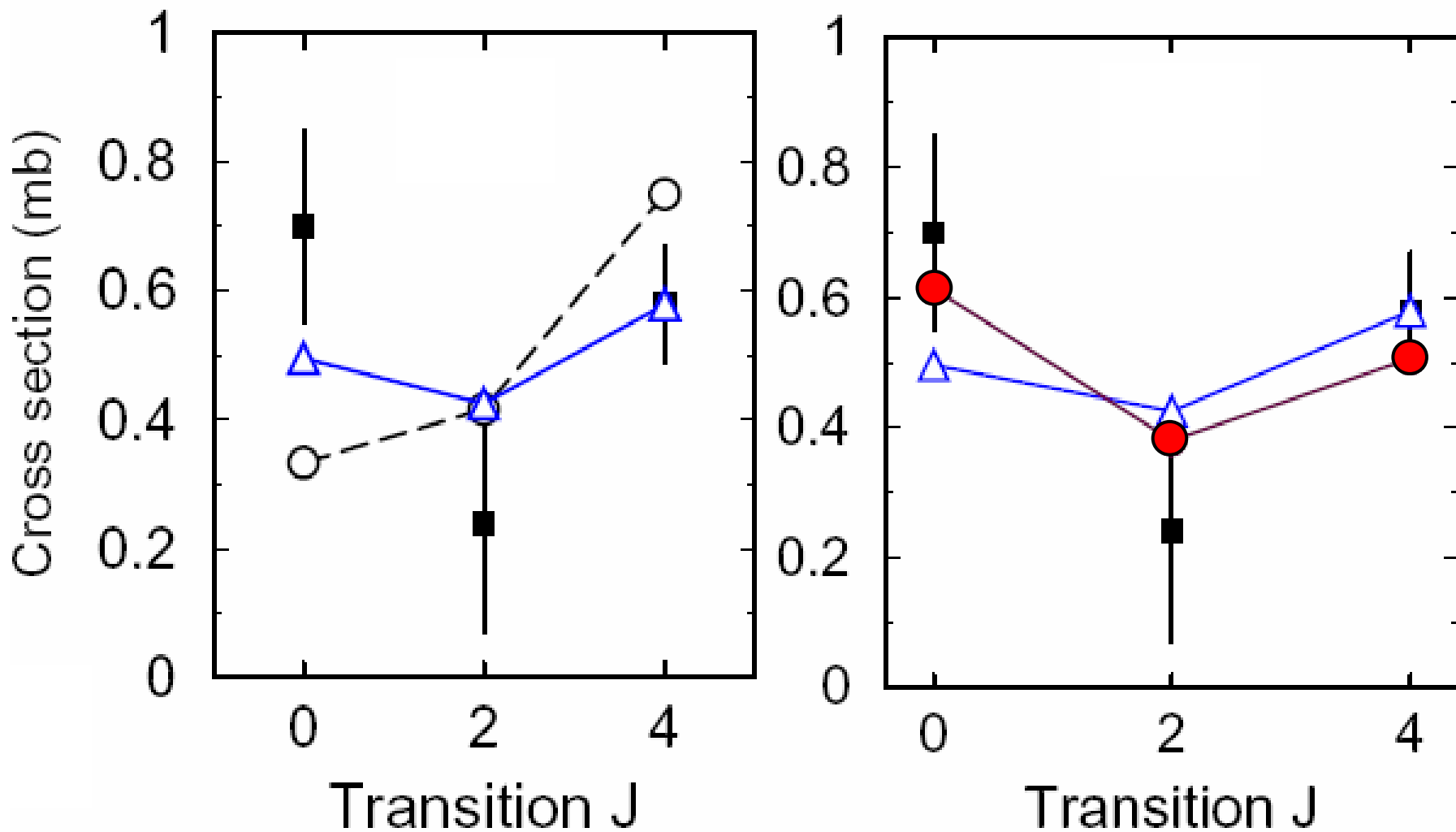
$^{48}\text{Ca}$

\* 3.580      3.560 fm (HF)

# Our sensitivity is not to the ANC



Correlated:  $^{28}\text{Mg} \rightarrow ^{26}\text{Ne}(0^+, 2^+, 4^+)$ , 82.3 MeV/u

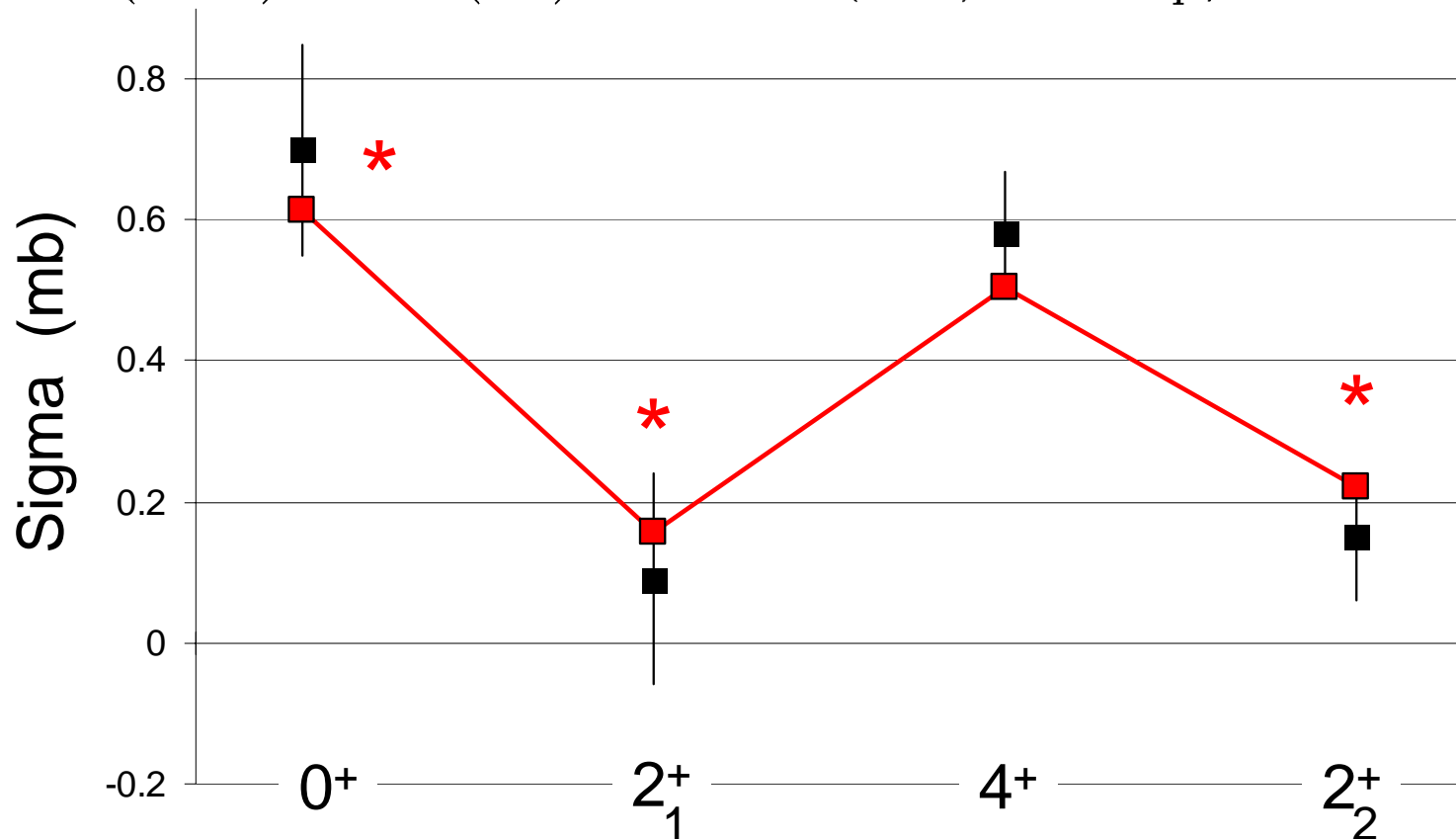


Data: D. Bazin et al., PRL **91** (2003) 012501

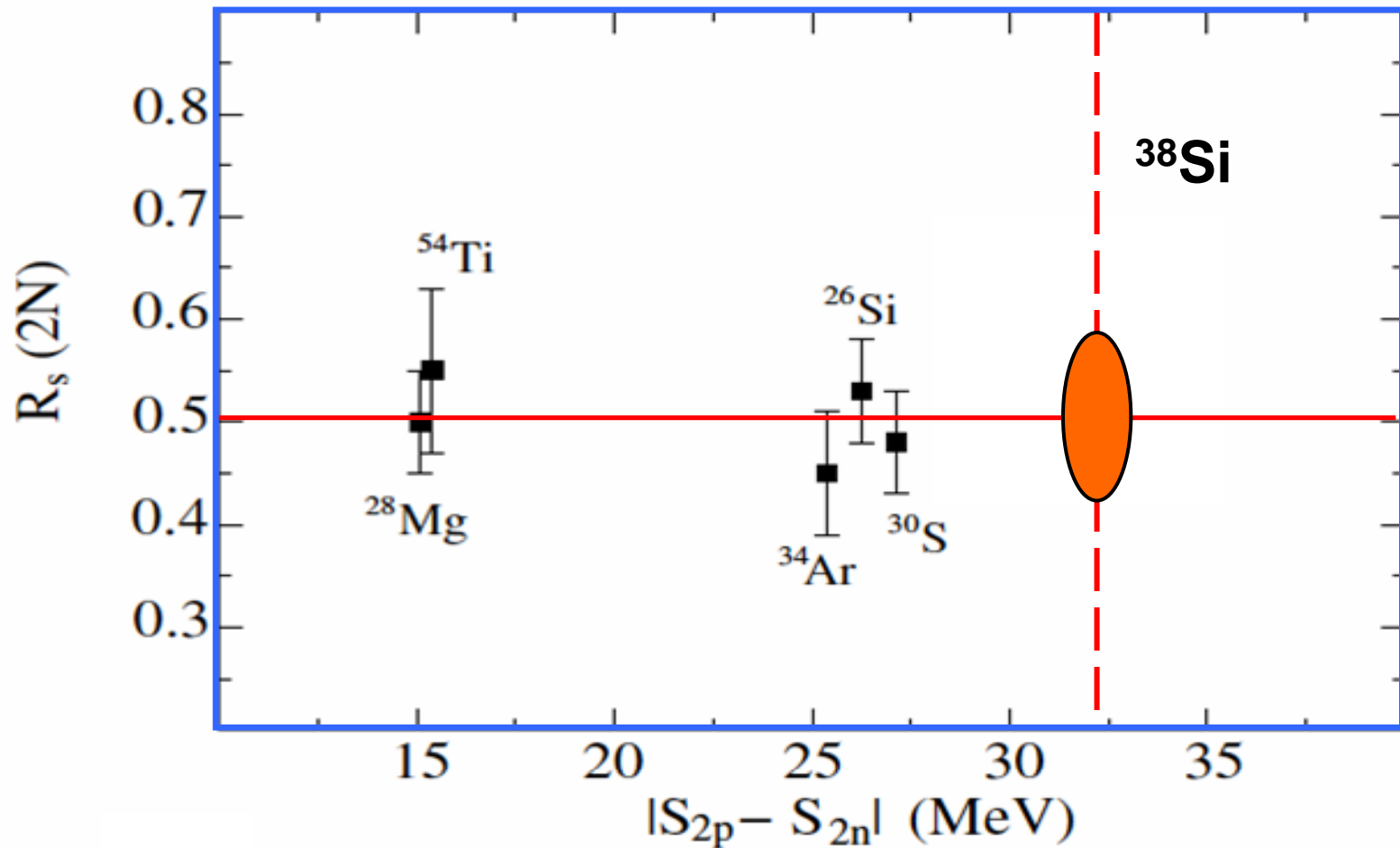
# Knockout cross sections – correlated case

$^{28}\text{Mg} \rightarrow ^{26}\text{Ne}(0^+, 2^+, 4^+, 2_2^+) \quad 82.3 \text{ MeV/u}$

$$\sigma_{inc}(-2p) = 1.50(10) \text{ mb}, \quad R_s(2N) = \sigma_{\text{exp}}/\sigma_{\text{th}} = 0.52(4)$$



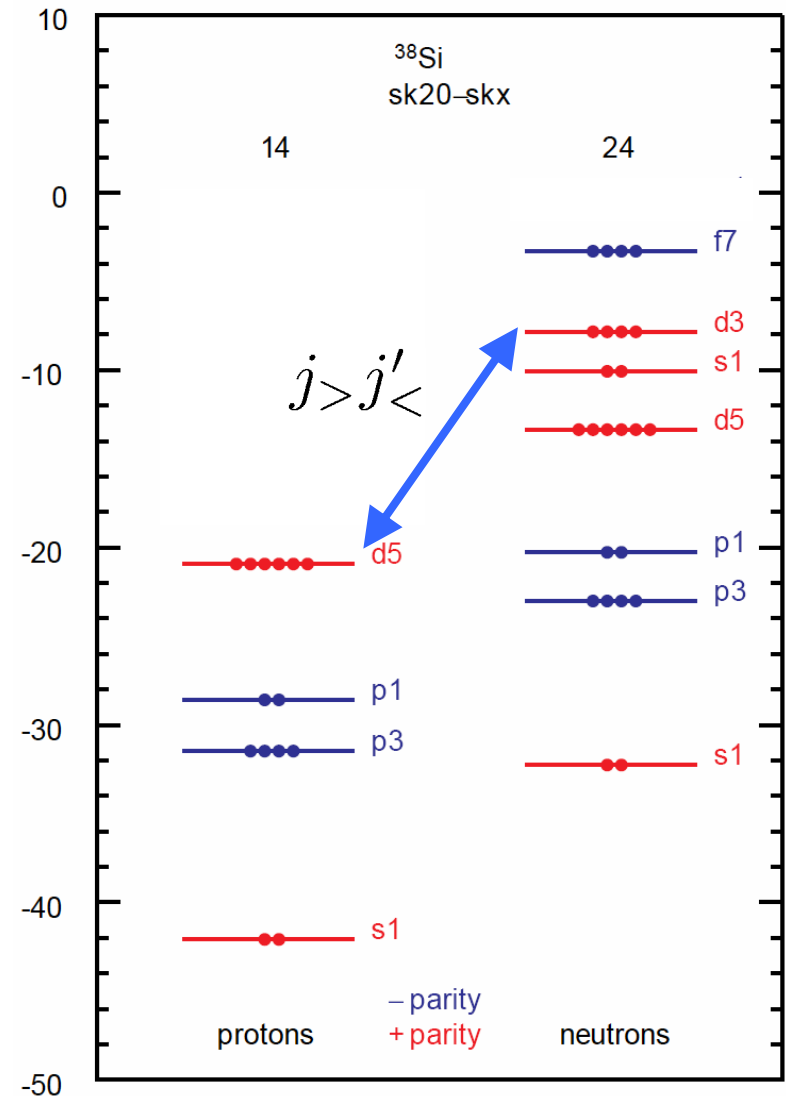
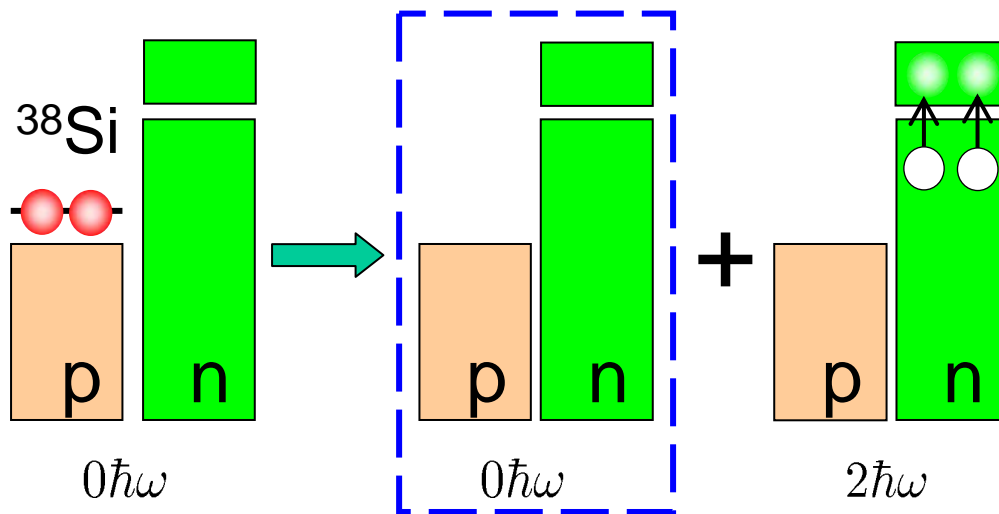
# Ratio of measured to calculated cross sections



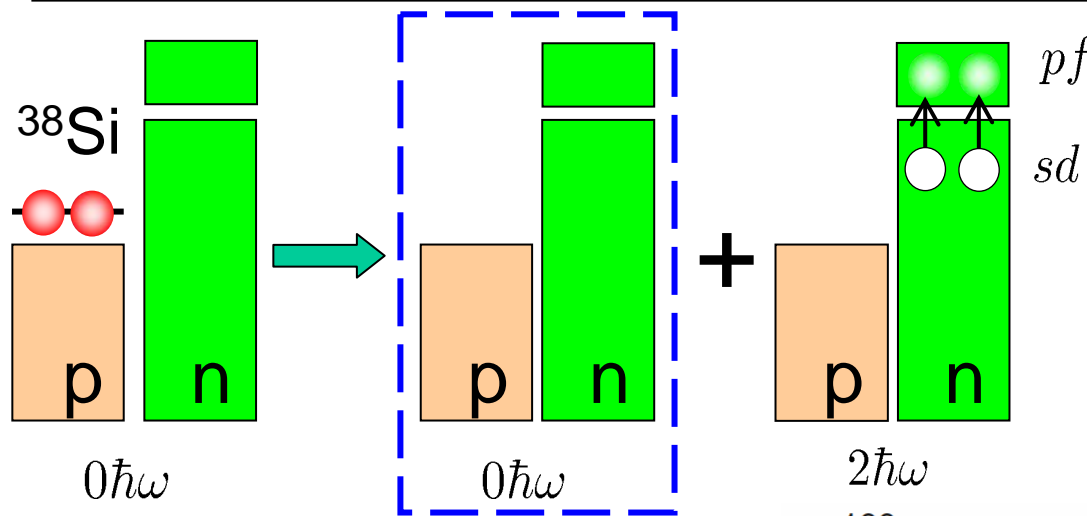
J.A. Tostevin and B.A. Brown, PRC **74** 064604 (2006), PRC **70** 064602 (2004)  
 Figure: A. Gade

# Extent of the Island of Inversion: to $^{36}\text{Mg}$ ?

- 1) Insufficient yield for, e.g. secondary beam inelastic scattering
- 2) Parent for beta decay,  $^{37}\text{Na}$ , is particle unbound
- 3) can use 2p removal from n-rich (sd-shell) parent,  $^{38}\text{Si}$



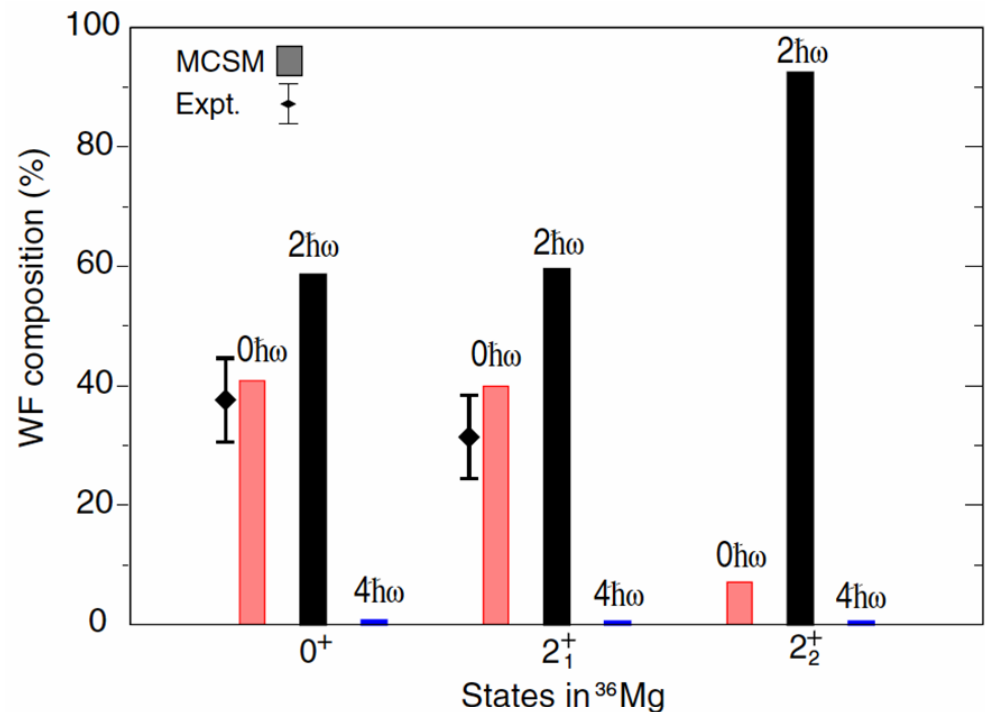
# $^{38}\text{Si}(-2p)$ to $^{36}\text{Mg}$ – shell model and interactions?



Monte-Carlo shell model calculations: SDPF-M interaction of Utsono, Otsuka *et al.*

Measured cross sections and those calculated assuming population of the  $0\hbar\omega$  components of the final states by the direct  $2p$  knockout reaction mechanism

A. Gade *et al.*, PRL **99**, 072502 (2007)



Status: we are able to

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- calculate two-nucleon removal within a parameter free model – using specific shell model and systematic mean-field (spherical HF) structure information to set all important interaction and bound states ranges
- observe systematic renormalisation of (1N and) 2N strength –not yet understood quantitatively but already allows the identification of structure effects beyond these systematics
- have identified spectroscopic value of momentum distributions of -2N reactions and have a more complete calculation available