Spectroscopic factors and final state interactions in quasi-free electron-nucleus scattering

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Outline

- ⋆ Single particle dynamics in interactig many-body systems:
  overlaps & spectral functions

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- ⋆ Nuclear transparency
- ⋆ Summary & prospects

Single particle properties in interacting many-body systems

- ⋆ Overlaps are well (and *uniquely*) defined quantities for interacting many-body systems

$$\chi_n(\mathbf{r}_1) = \int d^3r_2 \ldots d^3r_\mathrm{A} \; \Psi_n^{\mathrm{A}-1}(\mathbf{r}_2 \ldots \mathbf{r}_A)^\dagger \Psi_0^{\mathrm{A}}(\mathbf{r}_1 \ldots \mathbf{r}_A)$$

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- ⋆ In momentum space they are directly related to the spectral function, yielding the energy-momentum probability distribution of hole states

$$P(\mathbf{k}, E) = \sum_n |\hat{\chi}_n(\mathbf{k})|^2 \delta(E - E_n + E_0)$$

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- ⋆ If $|\Psi_n^{\mathrm{A}-1}\rangle$ is a bound state χ_n carries information on single particle dynamics
- ⋆ Within the mean field picture $\chi_n \to \phi_n^{\mathrm{MF}}$, the n-th single particle orbital

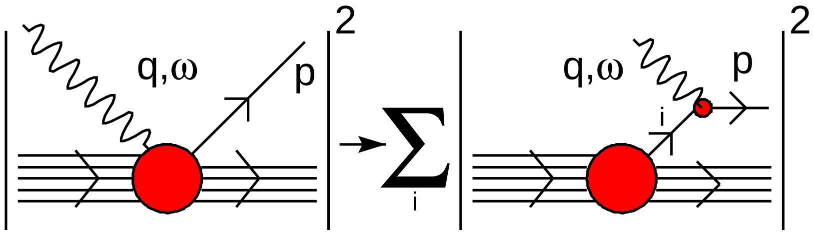
Spectroscopic factors

- ⋆ *In principle* the spectroscopic factors

$$Z_n = \int \frac{d^3k}{(2\pi)^3} \; |\hat{\chi}_n(\mathbf{k})|^2$$

can be extracted from the (non trivial !) analysis of the $(e, e'p)$ x-section

- ⋆ In Plane Wave Impulse Approximation (PWIA)



$$\sigma_{eA} = K\sigma_{ep} \, P(\mathbf{p} - \mathbf{q}, \omega - T_p) \ ,$$

where σ_{ep} is the elctron scattering cross section off a *bound, moving* nucleon and K is a kinematical factor

Spectroscopic factors in uniform nuclear matter

- ⋆ As momentum is a good quantum number, the spectral function at $|\mathbf{k}| < k_F$ exhibits only one peak
- ⋆ The spectroscopic factor is defined as

$$Z_k = \left| \langle \Phi^{1h}_{\mathbf{k}} | a_{\mathbf{k}} | \Psi_0 \rangle \right|^2 \ ,$$

where $|\Phi^{1h}_{\mathbf{k}}\rangle$, is the *one-hole* ((A-1)-nucleon) state carrying momentum $\mathbf{k}$

- ⋆ Note: Z_k *does not* coincide with the occupation number of the state $|\Phi^{1h}_{\mathbf{k}}\rangle$, $n(\mathbf{k})$, given by

$$n(\mathbf{k}) = \langle \Psi_0 | a^\dagger_{\mathbf{k}} a_{\mathbf{k}} | \Psi_0 \rangle = \sum_n |\langle \Phi^n_{\mathbf{k}} | a_{\mathbf{k}} | \Psi_0 \rangle|^2$$

where $\{|\Phi^n_{\mathbf{k}}\rangle\}$, is the complete set of (A-1)-nucleon states of momentum $\mathbf{k}$

- ⋆ $P(\mathbf{k}, E)$, Z_k and $n(\mathbf{k})$ of nuclear matter at equilibrium density ($\rho = 0.16$ fm^{-3}) have been calculated using realistic a realistic nuclear hamiltonian

$$H_A = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

$$\langle V_{ijk} \rangle \ll \langle v_{ij} \rangle$$

  - ▷ v_{ij} : π exchange + phenomenological short and intermediate range interaction. From fits to deuteron properties and $\sim$ 4000 NN scattering phase-shifts
  - ▷ V_{ijk} : Fujita-Miyazawa two-π exchange + phenomenological repulsive part. Needed to reproduce the empirical equilibrium density of nuclear matter and the measured binding energies of the three-nucleon bound states

Correlated Basis Function (CBF) perturbation theory

- ⋆ Replace the set of independent particle model states (Fermi gas, FG, in nuclear matter) with the *correlated* states

$$|n) = \frac{F|n_{\rm FG}\rangle}{\langle n_{\rm FG}|F^\dagger F|n_{\rm FG}\rangle^{1/2}}$$

- ⋆ The correlation operator F, whose structure reflects the structure of the interaction potential, is determined by minimization of the ground state expectation value of the nuclear hamiltonian

$$E_0^V = (\Psi_0|H|\Psi_0)$$

- ⋆ The correlated states are orthogonalized through a transformation that preserves diagonal matrix elements

$$|n) \to |n\rangle = \widehat{T}|n) \ \ , \ \ (n|H|n) = \langle n|H|n\rangle$$

- Split the hamiltonian acording to

$$H = H_0 + H_I$$

$$\langle m|H_0|n\rangle = \delta_{mn}\langle m|H|n\rangle \;\; , \;\; \langle m|H_I|n\rangle = (1-\delta_{mn})\langle m|H|n\rangle$$

- If correlated states have large overlaps with the eigenstates of the hamiltonian the matrix elements of H_I are small. Perturbation expansions are expected to be rapidly convergent.
- The nuclear matter spectral function is obtained starting from

$$P(\mathbf{k},E) = \frac{1}{\pi}\,\mathrm{Im}\,\langle\Psi_0|\frac{1}{H - E_0 - E - i\eta}|\Psi_0\rangle \; ,$$

expanding both $(H - E_0 - E - i\eta)^{-1}$ and $|\Psi_0\rangle$

⋆ Defining $\Delta E_0 = E_0 - E_0^V$ one finds

$$\frac{1}{H - E_0 - E - i\eta} = \frac{1}{H_0 - E_0^V - E - i\eta} \sum_m (-)^m \left(\frac{H_I - \Delta E_0}{H_0 - E_0^V - E - i\eta} \right)^m$$

$$|\Psi_0\rangle = \sum_m (-)^m \left(\frac{H_I - \Delta E_0}{H_0 - E_0^V} \right)^m |0\rangle$$

⋆ Calculations include correlated one hole and two hole-one particle intermediate states. The resulting $P(\mathbf{k}, E)$ can split into quasiparticle and background (i.e. *correlation*) contributions, $P_{QP}(\mathbf{k}, E)$ and $P_B(\mathbf{k}, E)$

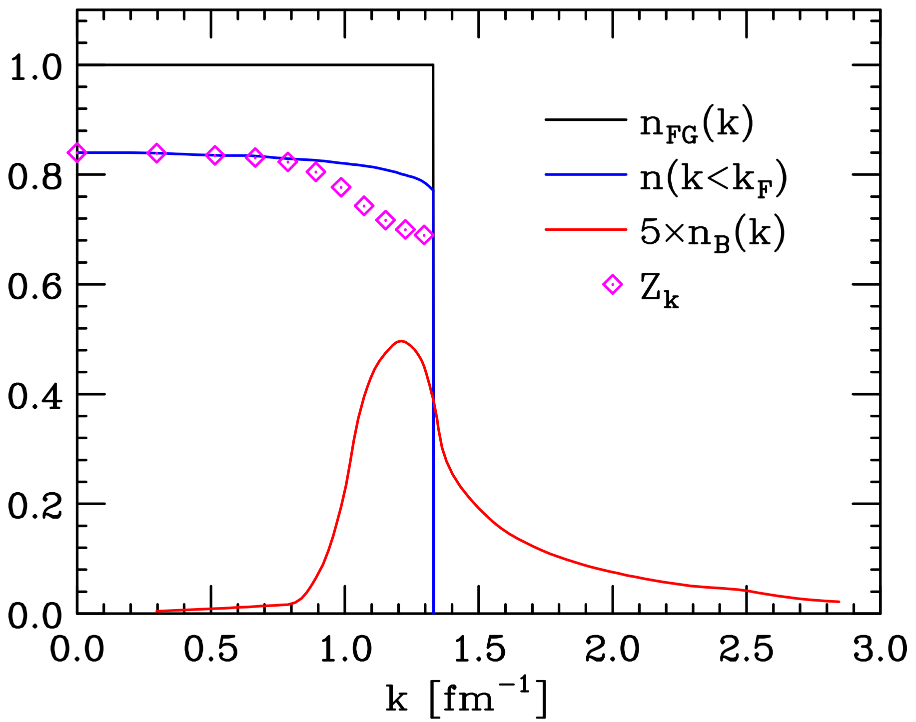
$$P(\mathbf{k}, E) = \frac{1}{\pi} \frac{Z_k^2 \,\mathrm{Im}\, \Sigma(\mathbf{k}, \epsilon_k)}{(E - \epsilon_k)^2 + [Z_k \mathrm{Im}\, \Sigma(\mathbf{k}, \epsilon_k)]^2} + P_B(\mathbf{k}, E)$$

where $\epsilon_k = k^2/2m + \mathrm{Re}\, \Sigma(\mathbf{k}, \epsilon_k)$, $\Sigma(\mathbf{k}, E)$ being the nucleon self energy

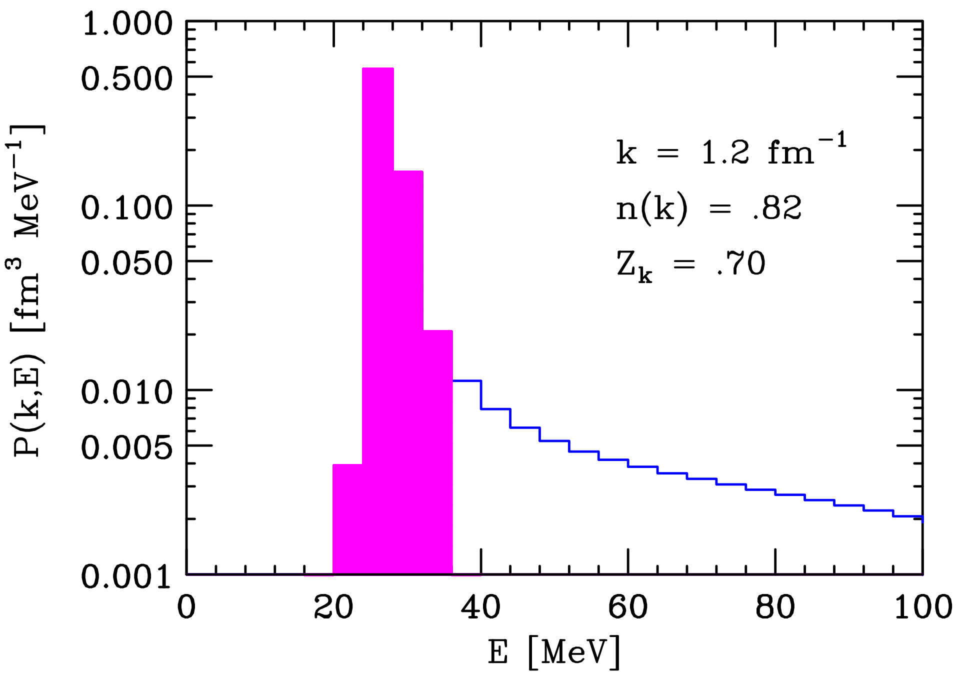
⋆ The momentum distribution can also be split into QP and B contributions

$$n(\mathbf{k}) = \int dE\, P(\mathbf{k}, E) = Z_k + \int dE\, P_B(\mathbf{k}, E) = Z_k + n_B(\mathbf{k})$$

⋆ Z_k is discontinuous at $|\mathbf{k}| = k_F$ and vanishes at $|\mathbf{k}| > k_F$, while $n_B(\mathbf{k})$ is continuous across the Fermi surface (Benhar, Fabrocini & Fantoni PRC 41(90)R24)



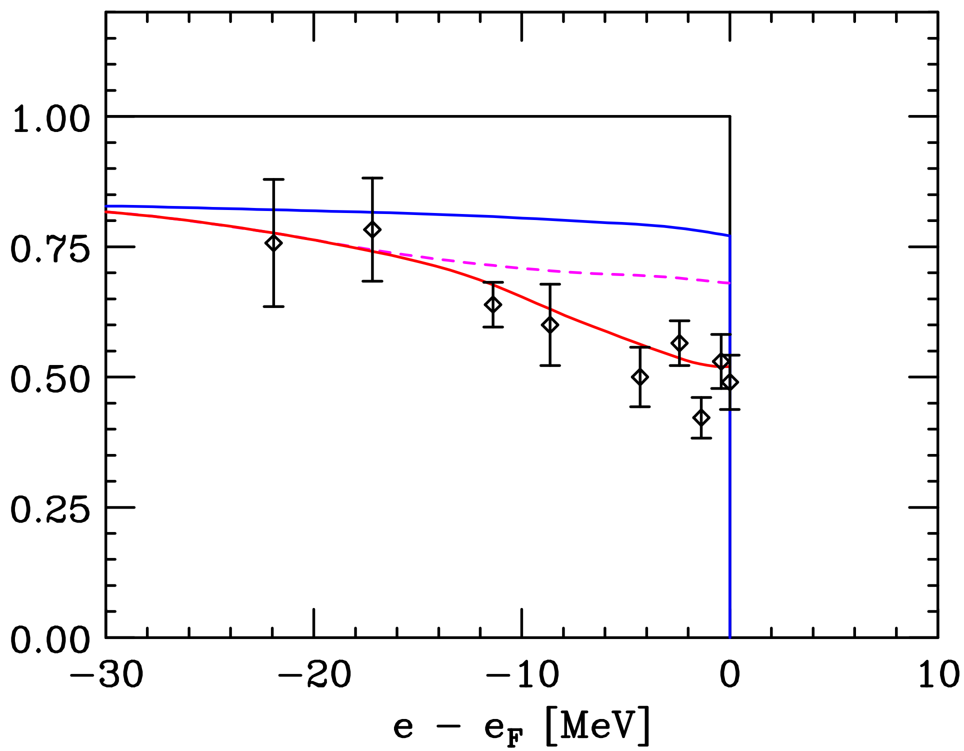
⋆ The difference between Z_k and $n(k)$ naturally emerges from the analysis of the spectral function at fixed $|\mathbf{k}| < k_F$



▷ integration over the peak region yields Z_k

▷ integration over the whole energy range yields $n(k)$

- ⋆ High resolution $(e, e'p)$ experiments measure Z_k
- ⋆ Comparison to NIKHEF-K data (Quint, Ph.D. Thesis, 1988).



- ▷ Dashed magenta line: nuclear matter results
- ▷ Solid red line: *estimated* surface effects included

  (Benhar, Fabrocini & Fantoni PRC 41(90)R24)

Many-body theory of final state interactions (FSI)

⋆ Consider the transition matrix element of the process

$$e + A \to e' + p + (A-1)_\alpha$$

$$M_\alpha(\mathbf{p},\mathbf{q}) = \langle \Psi^{(-)}_{\alpha\mathbf{p}} | \mathbf{J}(\mathbf{q}) | \Psi_0 \rangle$$

⋆ The target ground-state wave function satisifes the Scrödinger equation

$$H_A |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

⋆ Isolate the contributions responsible for final state interactions in the nuclear hamiltonian

$$H_A = H_0 + H_{FSI} \quad , \quad H_0 = T_1 + H_{A-1}$$

where T_1 is the kinetic energy of the struck nucleon

⋆ Scattering state

$$|\Psi^{(-)}_{\alpha\mathbf{p}}\rangle = \Omega^{(-)}_{\mathbf{p}}|\Phi_{\alpha\mathbf{p}}\rangle$$

$$|\Phi_{\alpha\mathbf{p}}\rangle = |\mathbf{p}\rangle \otimes |\varphi_\alpha\rangle$$

$$T_1|\mathbf{p}\rangle = E_p|\mathbf{p}\rangle \;\;,\;\; H_{A-1}|\varphi_\alpha\rangle = E^\alpha_{A-1}|\varphi_\alpha\rangle$$

⋆ Distortion (Möller) operator

$$\Omega^{(-)}_{\mathbf{p}} = \lim_{t\to\infty} \mathrm{e}^{iH_A t}\mathrm{e}^{-iH_0 t} = \lim_{t\to\infty} \widehat{T}\, \mathrm{e}^{-i\int_0^\infty dt' H_{FSI}(t')}$$

$$H_{FSI}(t) = \mathrm{e}^{iH_0 t} H_{FSI} \mathrm{e}^{-iH_0 t}$$

High energy (Glauber) approximation

(A) Eikonal approximation : the outgoing proton moves along a straight trajectory in the direction of $\mathbf{p}$, with constant velocity $\mathbf{v}$

(B) Frozen approximation : the spectator nucleons are seen as a collection of fixed scattering centers

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⋆ Under assumptions (A) & (B), the distortion operator can be rewritten in coordinate space as $(R \equiv \{\mathbf{r}_1, \ldots, \mathbf{r}_A\})$

$$\Omega_{\mathbf{p}}^{(-)}(R) = P_z \frac{1}{A} \sum_{i=1}^{A} \left[1 - \sum_{j>i} \Gamma_p(i,j) + \sum_{k>j>i} \Gamma_p(i,j)\Gamma_p(1,k) - \ldots \right]$$

⋆ The z-ordering operator P_z prevents the occurrence of backward scattering.

- ⋆ FSI interactions are driven by the coordinate space t-matrix Γ_p, related to the NN scattering amplitude f_p through

$$\Gamma_p(i,j) = \theta(z_j - z_i)\gamma_p(|\mathbf{b}_j - \mathbf{b}_i|)$$

$$\gamma_p(b) = -\frac{i}{2}\int \frac{d^2k_t}{(2\pi)^2}\, \mathrm{e}^{i\mathbf{k}_t\cdot\mathbf{b}}\, f_p(k_t)$$

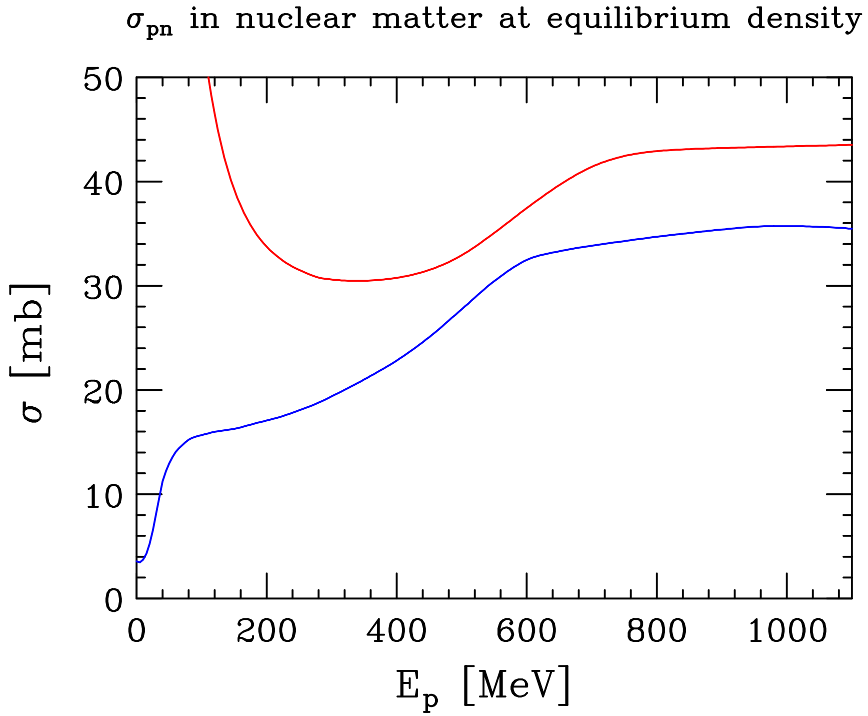
- ⋆ At large p, the *measured* free space $f_p(k_t)$ is generally parametrized in the for

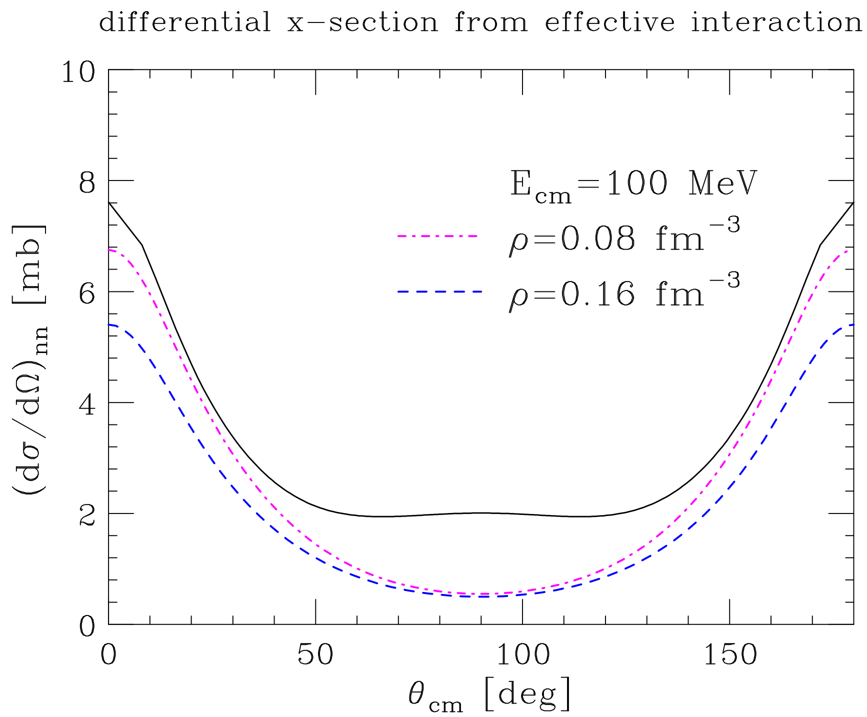
$$f_p(k_t) = i\,\sigma(1 - i\alpha)\mathrm{e}^{-\frac{1}{2}\frac{k_t^2}{B}}$$

where σ is the total NN cross section

⋆ Bad news: NN scattering in the nuclear medium expected to be appreciably modified by, e.g., Pauli blocking and dispersive effects

⋆ Good news: Medium modifications consistently calculable within many-body theory (Pieper & Pandharipande PRC 45(92)791, Benhar & Valli PRL 99(07)232501)



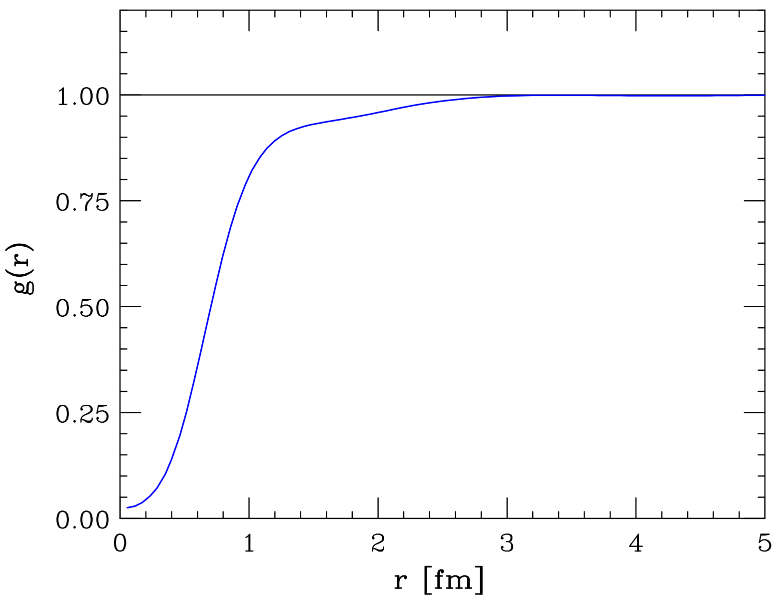


⋆ Local Density Approximation (LDA)

$$\Omega_{\mathbf{p}}^{(-)}(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})}\int d^3\mathbf{r}_1 \dots d^3\mathbf{r}_A |\Psi_0(\mathbf{r}_1 \dots \mathbf{r}_A)|^2$$

$$\times \frac{1}{A}\sum_{i=1}^{A}\left[1 - \sum_{j>i}\gamma_{\mathbf{p}}(\mathbf{b}_i - \mathbf{b}_i)\theta(z_i - z_j) + \dots\right]\delta(\mathbf{r} - \mathbf{r}_i)$$

$$g(\mathbf{r}_1, \mathbf{r}_2) = \frac{\rho(\mathbf{r}_1, \mathbf{r}_2)}{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)} \approx g_{NM}\left[|\mathbf{r}_1 - \mathbf{r}_2|, \rho_A\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)\right]$$



Transition amplitude including FSI effects

- ⋆ Recall: within PWIA

$$M_\alpha(\mathbf{p}-\mathbf{q}) = \int d^3r_1 \mathrm{e}^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}_1} \chi_\alpha(\mathbf{r}_1)$$

- ⋆ In the presence of FSI

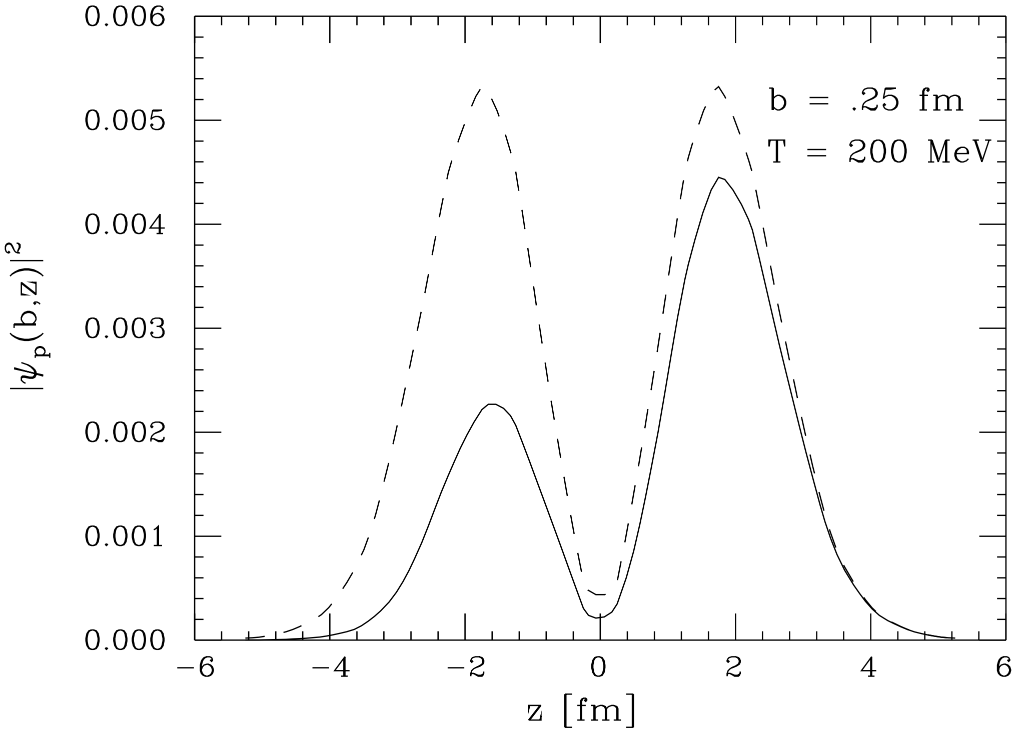
$$\chi_\alpha(\mathbf{r}) \to \psi_{\alpha\mathbf{p}}(\mathbf{r}) = \Omega^{(-)}_{\mathbf{p}}(\mathbf{r})\chi_\alpha(\mathbf{r})$$

- ⋆ Z_α is reduced by a transparency factor $T_{\alpha\mathbf{p}}$

$$Z_\alpha \to \widetilde{Z}_\alpha = \int \frac{d^3k}{(2\pi)^3} |\psi_{\alpha\mathbf{p}}(\mathbf{k})|^2 = T_{\alpha\mathbf{p}} Z_\alpha$$

- ⋆ The momentum distributions $|\psi_{\alpha\mathbf{p}}(\mathbf{k})|^2$ is shifted with respect to $|\chi_\alpha(\mathbf{k})|^2$

⋆ Knock out of a p-shell proton from oxygen: $|\psi_p^{\mathbf{p}}(\mathbf{r})|^2$ vs $|\chi_p(\mathbf{r})|^2$

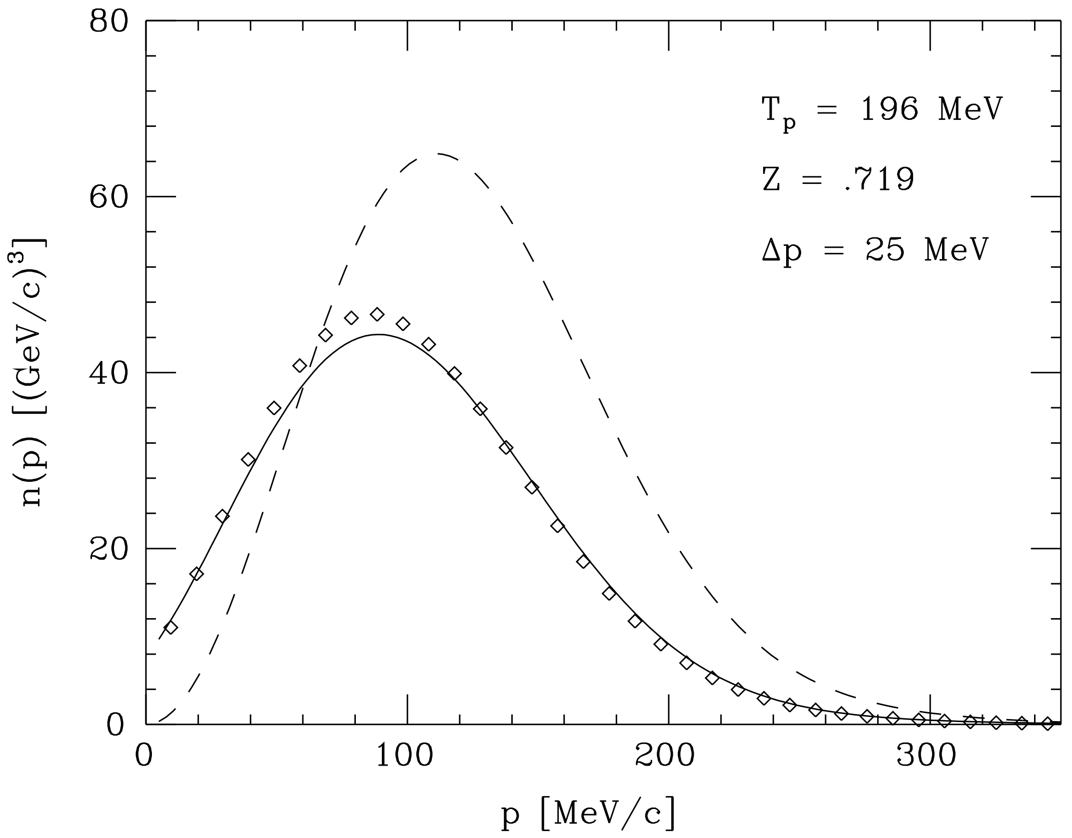


⋆ Spectroscopic factor and transparency

$$Z_p = \int d^3r\ |\chi_p(\mathbf{r})|^2 = .62 \quad , \quad T_p = \frac{1}{Z_p} \int d^3r\ |\psi_p^{\mathbf{p}}(\mathbf{r})|^2 = .72$$

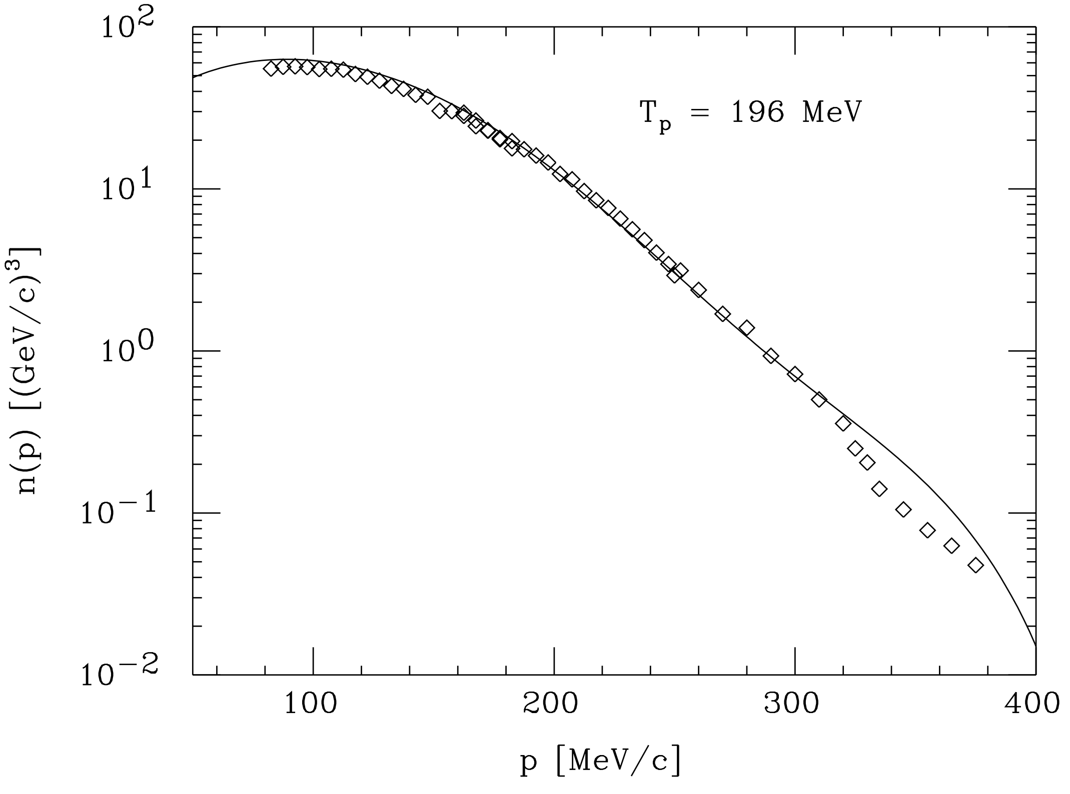
⋆ Momentum distribution:

$$|\widehat{\psi}_p^{\mathbf{p}}(|\mathbf{p}-\mathbf{q}|)|^2 \approx \widetilde{Z}_p|\chi_p(|\mathbf{p}-\mathbf{q}|+\Delta_p)|^2$$



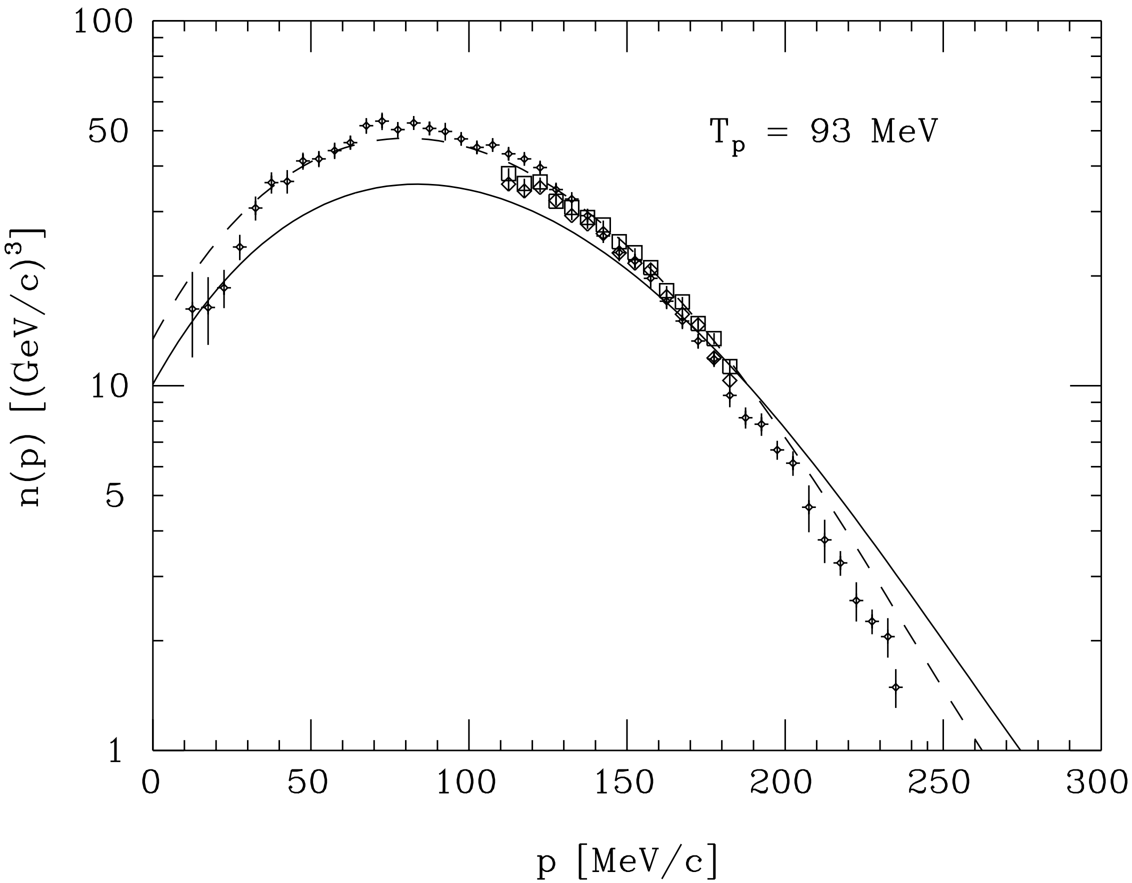
⋆ Mainz data: Blomqvist *et al* (PLB 344(95)85).

$$Z = 0.62 \quad , \quad \widetilde{Z}_{\mathbf{p}} = 0.72$$

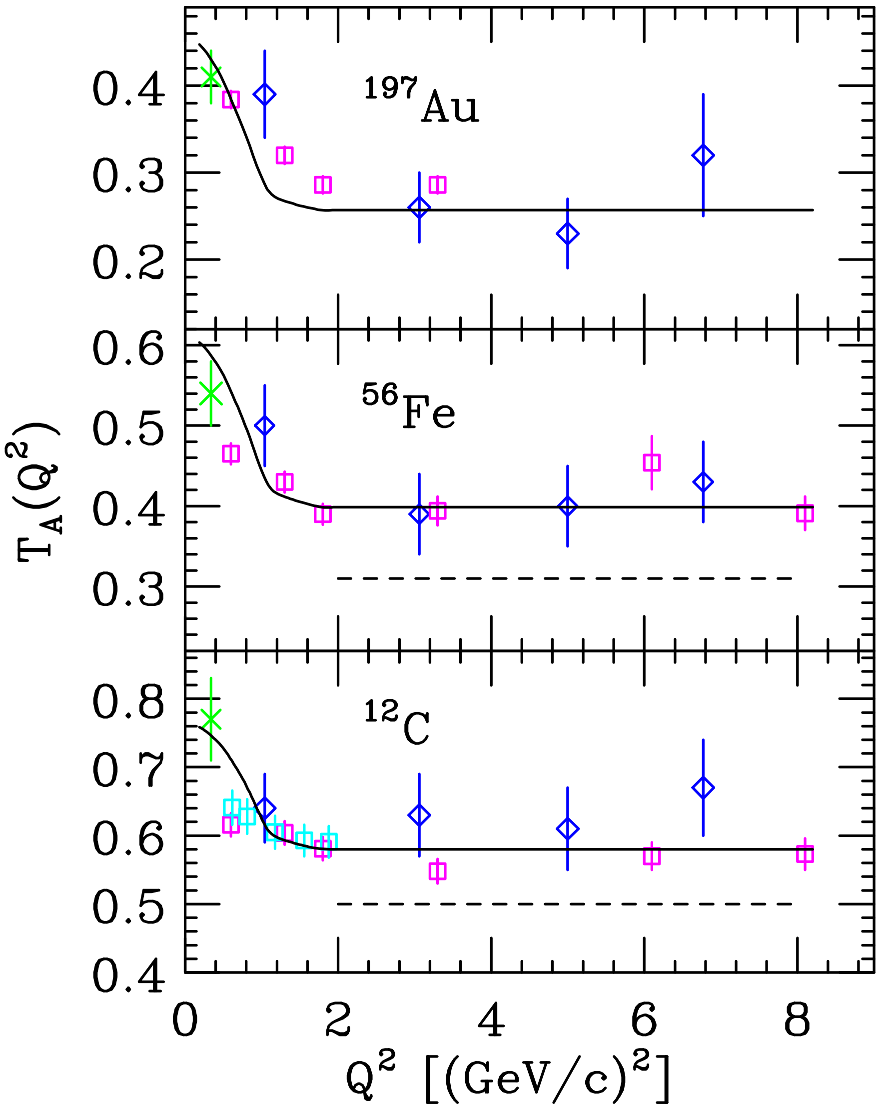


⋆ NIKHEF-K data: Leuschner *et al* (PRC 49(94)955)

- ▷ solid line: $\chi(\mathbf{r}) = \langle {}^{15}N(3/2)^{-}|a_{\mathbf{r}}|{}^{16}O\rangle \quad , \quad Z = 0.62$
- ▷ dashed line: $\chi(\mathbf{r}) = \sqrt{Z}\, \phi_{WS}(\mathbf{r}) \quad , \quad Z = 0.56$



Nuclear transparency (no FSI → $T_A \equiv 1$)



- Nuclear transparency obtained from

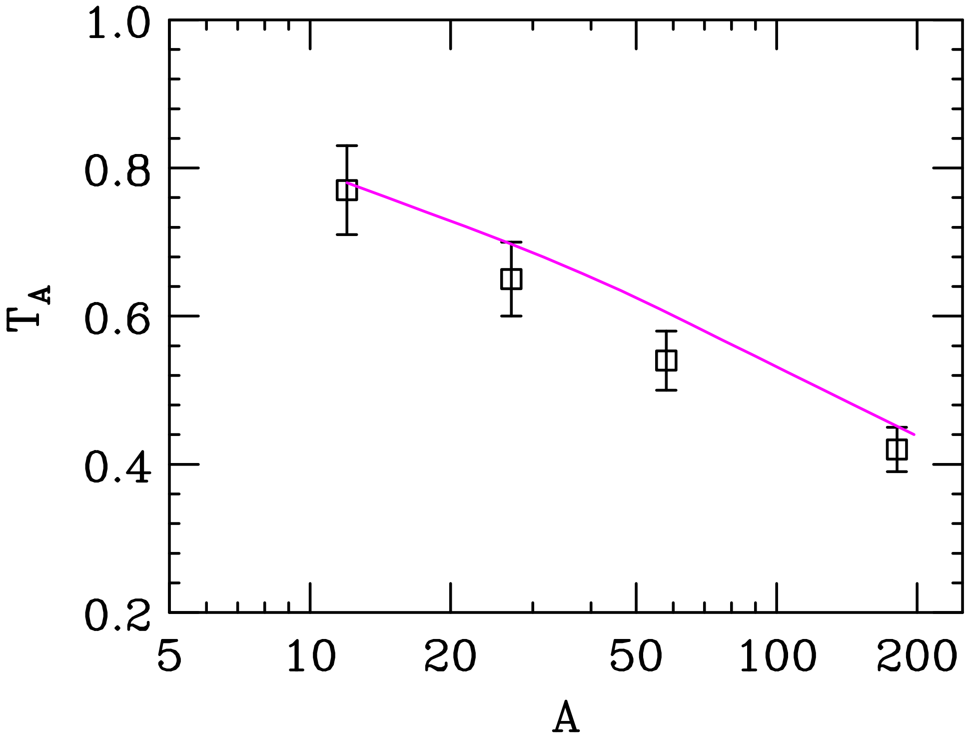
$$T_{\rm A} = \frac{1}{A}\int d^3r\;\rho_A(\mathbf{r})\;|\Omega_{\mathbf{p}}^{(-)}(\mathbf{r})|^2$$

compared to MIT-Bates, SLAC and JLab data (D. Rohe *et al* Phys. Rev. C 72(05)054602)

- Complicated pattern of correlation effects, leading to a sizable enhancement of the transparency

How low can the proton energy be ?

- ⋆ Compare theory to the A-dependence of nuclear transparency to a 200 MeV proton, measured at MIT



- ⋆ The high energy approximation appears to work down to surprisingly low energy

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Summary & prospects

- ⋆ Quasifree electron nucleus scattering experiments have provided a wealth of information on nuclear structure and dynamics
- ⋆ The spectroscopic factors extracted from $(e, e'p)$ data have clearly exposed the limits of the mean field picture and the importance of nucleon-nucleon correlations
- ⋆ Nuclear many-body theory, based on realistic dynamical models, provides a fully consistent theoretical framework for the analysis and the interpretation of the data