Dissipation in Multidimensional Quantum Tunneling and Subbarrier Fusion

A.B. Balantekin
University of Wisconsin-Madison

ECT* Workshop “Decoherence in QM Systems
April 2010
One-dimensional quantum tunneling is simple!
incident
What we do:

$$\hat{H} = \frac{P^2}{2M} + V(R) + H_0(q) + V_{\text{int}}(q, R)$$

- Barrier + an absorbing potential
- “environment”

Sometimes it is easier to think of an idealized problem...
Calculating the fusion cross-section:

\[
\hat{H} = \frac{P^2}{2M} + V(R) + H_0(q) + V_{\text{int}}(q, R)
\]

\[
\sigma(E) \propto \sum_f \left| \langle f \left| \frac{1}{E - \hat{H}} \right| i \rangle \right|^2
\]
Eigenchannels, assume $H_0 \ll E$
Eigenchannels, assume $H_0 \ll E$

$$\sigma(E) \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \frac{p^2}{2M} - V(R) - V_{\text{int}}(q, R)} \right| i \right\rangle \right|^2$$

$$\sigma \propto \sum_f \left| \left\langle f \left| \frac{1}{E - \frac{p^2}{2M} - V(R) - V_{\text{int}}(q, R)} \left( \int d^3q |q\rangle \langle q| \right) \right| i \right\rangle \right|^2$$

If \[ \sum_f |f\rangle \langle f| = 1 \]

$$\sigma_{\text{total}} = \int d^3q |\Psi_i(q)|^2 \sigma(\text{calc. w/ pot. } V(R) \oplus V_{\text{int}}(q, R))$$
Classical

Quantum Mechanical

\[ \frac{dT}{dE} \quad \text{Energy} \]

\[ T(E) \quad \text{Energy} \]
Subbarrier Fusion - Experimental Observables

\[ \sigma(E) = \sum_{\ell=0}^{\infty} \sigma_{\ell}(E), \]

\[ \langle \ell(E) \rangle = \frac{\sum_{\ell=0}^{\infty} \ell \sigma_{\ell}(E)}{\sum_{\ell=0}^{\infty} \sigma_{\ell}(E)}. \]

\[ \sigma_{\ell}(E) = \frac{\pi \hbar^2}{2\mu E} (2\ell + 1) T_{\ell}(E), \]
One-dimensional Model

\[ T_\ell(E) = \left[ 1 + \exp \left( \frac{2\mu}{\hbar^2} \int_{r_1}^{r_2} dr \left[ V_0(r) + \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2} - E \right]^{1/2} \right) \right]^{-1} \]

\[ T_\ell \simeq T_0 \left[ E - \frac{\ell(\ell + 1)\hbar^2}{2\mu R^2(E)} \right] \]

\[ E\sigma(E) = \pi R^2(E) \int_0^E dE' T_0(E') \]

\[ \frac{dT_0(E)}{dE} \sim \frac{1}{\pi R^2(E)} \frac{d^2}{dE^2} (E\sigma(E)) + O\left(\frac{dR}{dE}\right). \]
Classical versus Quantum Tunneling

Rowley, Satchler, and Stelson, 1997
What may be missing from coupled-channels calculations?

\[ \sigma \propto \sum_f |\langle f | G(E) | i \rangle|^2 = \langle i | G(E)^* G(E) | i \rangle \]

\[ \sum_n |n\rangle\langle n| = 1 \]

But in a practical coupled-channels calculation we have

\[ \sum_{\text{states included}} |n\rangle\langle n| < 1 \]
An outstanding question: Why is the diffuseness for both fusion and quasi-elastic scattering equal to 1.5 to 2 times the diffuseness for elastic scattering?
For asymmetric systems Coulomb force is relatively weaker; hence the tail of the nuclear potential can “turn over” the sum, forming the barrier at a relatively large separation:
On the other hand for symmetric systems Coulomb force is relatively stronger; hence it takes more of the nuclear potential to “turn over” the sum, forming the barrier at very close separations:
Another example: Neutrino Oscillations in fluctuating electron background

\[ H = \left( -\frac{\delta m^2}{4E} \cos 2\theta + \frac{G_F}{\sqrt{2}} [N_e(r) + N_e^r(r)] \right) \sigma_z + \left( \frac{\delta m^2}{4E} \sin 2\theta \right) \sigma_x \]

\[ \langle N_e^r(r) \rangle = 0 \]

\[ \langle N_e^r(r)N_e^r(r') \rangle = \beta^2 N_e(r)N_e(r') \exp(-|r - r'|/\tau_c) \]
Does the solar density fluctuate?

Solar data only

Fluctuations

$\beta = 0.000$

$\delta m_{21}^2 (\text{eV}^2)$

$\tan^2 \theta_{12}$
Solar + KamLAND

Fluctuations

$\beta = 0.000$

$\delta m^2_{21} (eV^2)$

$\tan^2 \theta_{12}$
Another example: Neutrino Oscillations in fluctuating electron background

\[
H = \left( -\frac{\delta m^2}{4E} \cos 2\theta + \frac{G_F}{\sqrt{2}} [N_e(r) + N_e^r(r)] \right) \sigma_z + \left( \frac{\delta m^2}{4E} \sin 2\theta \right) \sigma_x
\]

\[
\langle N_e^r(r) \rangle = 0
\]

\[
\langle N_e^r(r) N_e^r(r') \rangle = \beta^2 N_e(r) N_e(r') \exp(-|r - r'|/\tau_c)
\]

\[
\lim_\tau_{c \to \infty} \langle \hat{\rho}(r) \rangle = \frac{1}{\sqrt{2\pi} \beta^2} \int_{-\infty}^{+\infty} dx \exp\left(-x^2/(2\beta^2)\right) \hat{\rho}(r, x)
\]
A Simple Model of “Environment” - Balantekin & Takigawa

\[ H_0 = \hbar \omega \left( a_0^\dagger a_0 \right) + \sum_{i=1}^{m} \hbar \omega_i \left( b_i^\dagger b_i \right) + \hbar \kappa \sum_{i}^{m} \left( a_0^\dagger b_i + a_0 b_i^\dagger \right) \]

\[ H_{\text{int}} = \alpha_0 f(R) (a_0^\dagger + a_0) \]

Bogoliubov transformation

\[ \tilde{a}_j = \chi_{j1} a_0 + \sum_{i=1}^{m} \chi_{j,i+1} b_i \]

\[ \chi_{j1} = \left( 1 + \sum_{i=1}^{m} \frac{\kappa^2}{(\tilde{\omega}_j - \omega_i)^2} \right)^{-1/2} \]

\[ \chi_{j,i+1} = \frac{\kappa}{\tilde{\omega}_j - \omega_i} \chi_{j1} \]
\[ \tilde{H}_0 = \sum_{j=1}^{m+1} \hbar \tilde{\omega}_j \tilde{a}_j^\dagger \tilde{a}_j \]

Assume

\[ \omega_i = i \Delta, \quad i = 0, \pm 1, \pm 2, \ldots \]

\[ \frac{\pi \kappa \Delta}{\Delta} \gg 1 \]

Strength distribution

\[ J(\tilde{\omega}_j) = \frac{\chi_{j1}^2}{\Delta} = \frac{1}{2\pi} \frac{\Gamma}{(\tilde{\omega}_j - \omega_0)^2 + (\Gamma/2)^2} \]

Note that the strength distribution is not Ohmic: \( J(\omega) \neq \eta \omega \)
Subbarrier Fusion of $^9$Li with $^{70}$Zn

Data: Loveland, et al. PRC 74, 064609 (2006) measured using ISAC facility at TRIUMF

Calculation: Kocak and Balantekin using CCFULL
Could $^9$Li be capturing two neutrons from $^{70}$Zn prior to tunneling since $^{11}$Li is also stable?

Add a small potential to describe this two-neutron transfer:

$$V_{2n} = V_d \frac{d}{dr} \left( \frac{1}{1 + e^{\frac{r-R}{a}}} \right)$$
Questions

• For asymmetric systems the barrier is outside the region where nuclei touch. Multidimensional barrier penetration is conceptually well-defined. Do we really understand the fusion of such nuclei? What is the large diffuseness telling us?

• What happens when nuclei fuse at energies well-below the barrier? What physics does the very shallow potentials needed to fit the data mimic?

• Do we understand how we should theoretically formulate the fusion of unstable nuclei? What can we learn by studying fusion of nuclei off the line of stability?

• We need data for the fusion of exotic nuclei, both below and above the Coulomb barrier. Such data would open a new chapter in the study of multidimensional quantum tunneling.