Time-dependent approaches to quantum dynamics of many-body systems

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Contents

• Real-time propagation approaches to many-body quantum dynamics
• Time-dependent Schroedinger equation for nuclear fusion
• Time-dependent density-functional theory
  – Oscillator strength distribution in molecules
  – Dynamics under laser pulse
    • Molecular dissociation
    • Optical breakdown of dielectrics
Time-dependent Schroedinger equation

\[ i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = H\psi(\vec{r}, t) \]

Time-independent treatment

\[ \psi(\vec{r}, t) = \phi(\vec{r}) e^{-iEt/\hbar} \]
\[ H\phi(\vec{r}) = E\phi(\vec{r}) \]

Solve equation with scattering boundary condition

\[ \phi(\vec{r}) \rightarrow e^{ikz} + f(\Omega) \frac{e^{ikr}}{r} \]
\[ \frac{d\sigma}{d\Omega} = |f(\Omega)|^2 \]

Observables: cross section, etc.
Why time-dependent?

- Wave-packet dynamics provides intuitive picture
- No need for scattering boundary condition
  Advantage for complex systems: non-spherical potential, 3-body reaction, …

\[
\psi^{(+)}(\vec{r}) = \phi(\vec{r}) + \frac{1}{E + i\epsilon - H} V\phi(\vec{r}) = \phi(\vec{r}) + \frac{1}{i\hbar} \int_{0}^{\infty} dt e^{i(E + i\epsilon)t/\hbar} e^{-iHt/\hbar} V\phi(\vec{r})
\]

\[
\rightarrow \phi(\vec{r}) + f(\Omega) e^{ikr}/r
\]

- Full spectral information from single wave-packet dynamics

Two topics with time-dependent method

1. Three-body reaction of halo nuclei
2. TDDFT studies
Fusion reaction of halo nuclei

A real-time wave-packet method for three-body tunneling dynamics

Time-dependent approach to quantum dynamics in low-energy reaction
Fusion reaction in terms of flux loss inside a Coulomb barrier

*Time-independent* (radial) Schroedinger equation for l=0

\[ Eu(r,t) = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + V(r) + iW(r) \right] u(r,t) \]

Flux absorbed by \( W(r) \) represents fusion.

We need to take account of a boundary condition at \( r = r_\text{in} \), when solving the differential equation.

*Time-dependent* (radial) Schroedinger equation for l=0

\[ i\hbar \frac{\partial}{\partial t} u(r,t) = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + V(r) + iW(r) \right] u(r,t) \]

Use of wave packet does not require a boundary condition.
Wave packet dynamics of fusion reaction potential scattering with absorption inside a Coulomb barrier

Radial Schroedinger equation for \( l = 0 \)

\[
\frac{i\hbar}{\partial t} u(r, t) = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + V(r) + iW(r) \right] u(r, t)
\]

with incident Gaussian wave packet

\[
u(r, t_0) = \exp \left[ -ikr - \gamma (r - r_0)^2 \right]
\]

\(^{\text{10}}\text{Be} - ^{\text{208}}\text{Pb}\)

Flux absorbed by \( W(r) \) represents fusion.

Wave packet dynamics include scattering information for wide energy region. Then, how to extract reaction information for a fixed energy?
Extract static (fixed-E) information from wave-packet dynamics:
define energy distribution
\[ P_{a}(E) = \langle u_{a} | \delta(E - H) | u_{a} \rangle = \frac{1}{2\pi\hbar} \int_{0}^{\infty} dt e^{iEt/\hbar} \langle u_{a}(-\frac{t}{2}) | u_{a}(\frac{t}{2}) \rangle \]
Fusion probability

\[ P_{\text{fusion}}(E) = \frac{P_{\text{init}}(E) - P_{\text{final}}(E)}{P_{\text{init}}(E)} \]

Fusion probability for whole barrier region from single wave-packet calculation. No boundary condition required in the wave packet calculation.
Fusion probability of three-body reaction

\[ i\hbar \frac{\partial}{\partial t} \psi(R, r, t) = \left( -\frac{\hbar^2}{2\mu} \nabla_R^2 - \frac{\hbar^2}{2m} \nabla_r^2 + V_{nc}(r_{nc}) + V_{CT}(r_{CT}) + V_{nT}(r_{nT}) \right) \psi(R, r, t) \]

Initial incident wave

\[ \psi_j(R, r, t) = \sum_l \frac{u_l^j(R, r, t)}{Rr} P_l(\cos \theta) \]

Coulomb + Nuclear potential
Absorption => C-T fusion

- Elastic
- FUSION (Complete + Incomplete)
- Flux loss by absorption
- Breakup
- Transfer
Case (1): Tightly-bound projectile

3-body dynamics
Tightly-bound projectile ($E_b = -3.5\text{MeV}$)  
($n^{-10\text{Be}})-^{40}\text{Ca}$

Initial wave packet:

$$u_i(R, r, t_0) = \delta_{i0} \exp \left[-iKR - \gamma(R - R_0)^2 \right] u_0(r)$$

Head-on collision ($J=0$)

$$\rho(R, r, t) = \int d(cos \theta) |\psi(R, r, \theta, t)|^2$$

$$\rho(r, \theta, t) = \int dR |\psi(R, r, \theta, t)|^2$$
Enhancement of fusion probability at sub-barrier energies
Transfer probability and Q-value matching

$E_P = -3.5\text{MeV}$

Projectile

Target

$E_P \approx E_T$

Strong mixing of projectile-target orbitals, large transfer probability
energy-dependent barrier for fusion

$E_P < E_T$
fusion enhancement

$E_P \approx E_T$
energy-dependent barrier

$E_P > E_T$
fusion suppression

Change n-T potential depth
Case (2): Weakly-bound projectile (*Neutron-halo*)

- n-C orbital energy: -0.6 MeV (Halo)

\[ ^{11}\text{Be}(n+^{10}\text{Be})^{-208}\text{Pb} \]

head-on collision (J=0)

\[ \rho(R,r,t) = \int d(cos \theta) |\psi(R,r,\theta,t)|^2 \]

\[ \rho(r,\theta,t) = \int dR |\psi(R,r,\theta,t)|^2 \]
Fusion probability of neutron-halo nuclei is suppressed

Core incident energy decreases effectively by neutron breakup

\[ E_{\text{core}} \approx \frac{M_{\text{core}}}{M_{\text{core}} + M_n} E_{\text{projectile}} \]
Why different from other studies?

Conclusions of other studies

- Quantum calculations have been done using the discretized continuum channels.
  - Diaz-Torres & Thompson, PRC65 (2002) 024606
- Fusion was enhanced with a weakly-bound neutron at sub-barrier energies
- Nuclear coupling was important for an the fusion enhancement

We need to include high-partial waves for n-\(^{10}\)Be motions.
The low-partial-wave truncation leads to an opposite conclusion!
Fusion Cross Section of $^{11}$Be

Fusion probability is hindered by the presence of the halo neutron

Experiment

Theory
Fusion cross section of $^6$He+$^{238}$U

Di-neutron model for $^6$He=$^4$He+(2n)


Calculation:
M. Ito, M. Ueda, T. Nakatsukasa, K. Yabana,
Fusion cross section of $^{15}$C+$^{144}$Sm

Fusion Cross Section (mb) vs $E_{c.m.}$ (MeV)

- $^{14}$C + $^{144}$Sm
- $^{15}$C + $^{144}$Sm
- 1-dim. by folding

$^{144}$Sm neutron

$r$, $R$
$^{11}\text{Be}-^{208}\text{Pb}$ fusion probability

Comparison between
Proton halo \((p^{-10}\text{Li})-^{208}\text{Pb}\)
and Neutron halo \((n^{-10}\text{Be})-^{208}\text{Pb}\)

Strong enhancement of Fusion Probability for Proton-Halo case
Summary

• Time-dependent approaches to quantum mechanical problems
  – Gross properties over a wide energy range
  – Continuum boundary condition
• Three-body nuclear fusion problem
  – Accurate calculation within the 3-body model
• Electronic TDDFT dynamics coupled with classical ionic dynamics
  – Dynamics under strong laser pulses suggest that the energy transfer from electrons to ions strongly depends on the pulse duration