The nuclear many-body problem: an open quantum systems perspective

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Coll: M. Assié, S. Ayik, Ph. Chomaz, G. Hupin, K. Washiyama

Trento, “Decoherence…” - April 2010
The nuclear many-body problem as an open quantum object

Generalities: Reduction of information

Environment

System

“Few” relevant degrees of freedom needs to be selected (System)

Illustrations discussed here

Fusion reactions: the role of open channels (discrete and continuous)
- System: collective space
- Env: intrinsic degrees of freedom

The nuclear many-body problem
- System: one-body observables
- Env: two-body and higher correlations
The nuclear many-body problem as an open quantum object

(i) Macroscopic reduction
The nuclear many-body problem as an open quantum object

(i) Macroscopic reduction

Other collective space: deformation, mass/charge asymmetry …
The nuclear many-body problem as an open quantum object

Open channels: discrete internal excitations

One of the difficulty is to treat both discrete and continuous channels in a common framework.
The nuclear many-body problem as an open quantum object

Stochastic semi-classical treatment of discrete channels

Excitation

Collective Motion + Coupling

\[ H = \frac{p^2}{2\mu} + \frac{l(l+1)h^2}{2\mu R^2} + V_C(R) + V_N(R, \Omega, \alpha_{i\lambda}) + \sum_{i=1}^{N-1} \sum_{\lambda=0}^{2} \left[ \frac{\Pi_{i\lambda}^2}{2D_{i\lambda}} + \frac{1}{2} C_{i\lambda} \alpha_{i\lambda}^2 \right] \]

Discrete Channels

Esbensen et al, PRL 41 (1978)

Initial Phase-space sampling of zero point motion

Classical dynamics of system+environment
With stochastic initial condition
The nuclear many-body problem as an open quantum object
Stochastic semi-classical treatment of discrete channels

Classical dynamics

\[ \frac{d R}{dt} = \frac{P}{\mu}, \]
\[ \frac{d P}{dt} = -\frac{d V_C(R)}{d R} - \frac{\partial V_N(R, \Omega, \alpha_{i\lambda})}{\partial R} + \frac{I(I + 1)\hbar^2}{\mu R^3} + \frac{\Pi_{i\lambda}}{D_{i\lambda}}, \]
\[ \frac{d \alpha_{i\lambda}}{dt} = \frac{\Pi_{i\lambda}}{D_{i\lambda}}, \]
\[ \frac{d \Pi_{i\lambda}}{dt} = -\frac{\partial V_N(R, \Omega, \alpha_{i\lambda})}{\partial \alpha_{i\lambda}} - C_{i\lambda} \alpha_{i\lambda}. \]

Ayik, Yilmaz, DL, PRC81 (2010)
The nuclear many-body problem as an open quantum object

Microscopic reduction

Mean-field: (DFT)

Self-consistent Mean-field

"Simple" Trial state:

\[ |\Phi_{HF}\rangle = \Pi a_\alpha^+ |0\rangle \]

Selection of few relevant degrees of freedom:

Courtesy to C. Simenel
Fusion reactions: macroscopic vs microscopic dynamics
Role of continuous channel: Disorder and Dissipation

Expected One-body origin of dissipation

- transfer of particle
- reflection of particles
Macroscopic reduction: dissipation


\[ \frac{dP}{dt} = \frac{dV}{dR} - \gamma(R) \frac{dR}{dt} \]

Dissipation

\[ V(R) \text{[MeV]} \]

R [fm]

Potential

Kinetic

Dynamical Reduction effect

Dissipation \rightarrow Internal Excitation

[Graphs and diagrams showing energy and radius relationships]
Fluctuations associated to dissipation

Application to fusion

Mean-field

\[ \frac{d}{dt} P = - \frac{d}{dR} U(R) - \gamma(R) \dot{R} \]

Mean-field+Initial fluct.

\[ \frac{d}{dt} p^\lambda = - \frac{d}{dR^\lambda} U(R^\lambda) - \gamma(R^\lambda) \dot{R}^\lambda + \xi^\lambda_P(t) \]

\[ \xi^\lambda_P(t) \xi^\lambda_P(t') = 2\delta(t - t') D_{PP}(R) \]
What next?

Semi-classical Phase-space dynamics

Add standard Dissipation (Markovian/Non-Markovian)

Add quantum Fluctuations associated To discrete channels

Incoherent Channels

Microscopic one-body dynamics

Coherent Channels

Semi-classical Phase-space dynamics
Part II
Mapping many-body systems To
Open quantum systems
D. Lacroix et al, Progress in Part. and Nucl. Phys. 52 (2004)

**Dynamics beyond mean-field**

**Projection technique**

**Short time evolution**

\[
\frac{i\hbar}{dt}\rho_1 = [h_{MF}, \rho_1] + T r_2 [v_{12}, C_{12}]
\]

\[
\frac{i\hbar}{dt}\rho_{12} = [h_{MF}(1) + h_{MF}(2), \rho_{12}]
+ (1 - \rho_1)(1 - \rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2v_{12}(1 - \rho_1)(1 - \rho_2)
\]

\[
C_{12} = \rho_{12} - (\rho_1\rho_2)_{A}
\]

**Approximate long time evolution + Projection (Nakajima-Zwanzig)**

\[
\frac{i\hbar}{dt}\rho_1 = [h_{MF}, \rho_1] + T r_2 [v_{12}, C_{12}]
\]

with

\[
C_{12}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s) ds + \delta C_{12}(t)
\]

- Projected two-body effect
- Propagated initial correlation

**Dissipation (Extended TDHF)**

\[
\frac{i\hbar}{dt}\rho = [h_{MF}, \rho] + K(\rho)
\]

**Dissipation and fluctuation**

\[
\frac{i\hbar}{dt}\rho = [h_{MF}, \rho] + K(\rho) + \delta K(\rho)
\]

**Random initial condition**
\[ i\hbar \frac{\partial}{\partial t} \rho_1 = [h_1[\rho], \rho_1] + \frac{1}{2} \text{Tr}_2 [\tilde{v}_{12}, C_{12}] \]

with

\[ C_{12}(t) = -i \int_{t_0}^{t} U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s) ds + \delta C_{12}(t) \]

\[ (1 - \rho_1)(1 - \rho_2)v_{12} \rho_1 \rho_2 - \rho_1 \rho_2 v_{12} (1 - \rho_1)(1 - \rho_2) \]

Non-Markovian master equation

\[ \frac{d}{dt} n_\lambda(t) = \int_{t_0}^{t} dt' \left\{ \tilde{n}_\lambda(t') W_\lambda^+(t, t') - n_\lambda(t') W_\lambda^-(t, t') \right\} \]

Example: two interacting fermions in 1dimension

Non-Markovian dynamics beyond mean-field application to collective motion

Giant Quadrupole resonances

Quadrupole moment vs. time (fm/c)

GQR in lead

$^{208}$Pb
exp. (p,p')

Mean energy is OK

Damping (dissipation) and fragmentation is missed

Incorporate dissipation in many-body system

Not so easy to use in Large amplitude Collective motion
Markovian limit, quantum-diffusion and stochastic Schrödinger Equation

GOAL: Restarting from an uncorrelated state  \( D = \left| \Phi_0 \right> \left< \Phi_0 \right| \) we should:

1. have an estimate of  \( D = \left| \Psi(t) \right> \left< \Psi(t) \right| \)

2. interpret it as an average over jumps between “simple” states

Weak coupling approximation: perturbative treatment

\[ |\Psi(t')\rangle = |\Phi(t')\rangle - \frac{i}{\hbar} \int \delta v_{12}(s) |\Phi(s)\rangle \, ds - \frac{1}{2\hbar^2} T \left( \int \int \delta v_{12}(s) \delta v_{12}(s') \, ds \, ds' \right) |\Phi(s)\rangle \]

Residual interaction in the mean-field interaction picture

Statistical assumption in the Markovian limit:

We assume that the residual interaction can be treated as an ensemble of two-body interaction:

\[
\begin{align*}
\delta v_{12}(s) &= 0 \\
\delta v_{12}(s) \delta v_{12}(s') &\propto \delta v_{12}^2(s) e^{-|s-s'|^2/2r^2}
\end{align*}
\]
Hypothesis: \( \tau \ll \Delta t \ll \tau_{\text{2coll}} \)

Average Density Evolution:

\[
\Delta \bar{D} = \frac{\Delta t}{i\hbar} [H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2} [\delta v_{12}, [\delta v_{12}, D]]
\]
One-body density
Master equation
step by step

Initial simple state

\[ D = |\Phi\rangle \langle \Phi| \]

\[ \rho = \sum_\alpha |\alpha\rangle \langle \alpha| \]

2p-2h nature of the interaction

Separability of the interaction

\[ v_{12} = \sum_\lambda O_\lambda(1)O_\lambda(2) \]

Dissipation: link between Extended TDHF and Lindblad Eq.

\[ \Delta D = \frac{\Delta t}{i\hbar} [H_{MF}, D] - \frac{\tau \Delta t}{2\hbar^2} [\delta v_{12}, [\delta v_{12}, D]] \]

\[ i\hbar \frac{d}{dt} \rho = [h_{MF}, \rho] - \frac{\tau}{2\hbar^2} \mathcal{D}(\rho) \]

with

\[ \langle j | \mathcal{D} | i \rangle = \left\langle \left[ a_i^+ a_j, \delta v_{12} \right], \delta v_{12} \right\rangle \]

\[ \mathcal{D}(\rho) = Tr_2 [v_{12}, C_{12}] \]

with

\[ C_{12} = (1 - \rho_1)(1 - \rho_2)v_{12}\rho_1\rho_2 - \rho_1\rho_2v_{12}(1 - \rho_1)(1 - \rho_2) \]

\[ \mathcal{D}(\rho) = \sum_k \gamma_k (A_k A_k \rho + \rho A_k A_k - 2A_k \rho A_k) \]

- Dissipation contained in Extended TDHF is included
- The master equation is a Lindblad equation
SSE on single-particle state:

\[ d|\alpha\rangle = \left\{ \frac{dt}{\hbar} h_{MF}(\rho) + \sum_k dW_k(1 - \rho) A_k - \frac{d\tau}{\hbar^2} \sum_k \gamma_k \left[ A_k^2 \rho + \rho A_k \rho A_k + 2A_k \rho A_k \right] \right\} |\alpha\rangle \]

\[ \text{with } dW_k dW_{k'} = -\frac{dt \tau}{\hbar^2} \gamma_k \delta_{kk'} \]

The numerical effort is fixed by the number of \( A_k \)

1D bose condensate with gaussian two-body interaction

N-body density: \( D = |N : \alpha\rangle \langle N : \alpha| \)
Towards Exact stochastic methods for N-body and Open systems
Self-interacting vs Open Quantum systems
Approximate and exact Quantum jump

\[ H = H_S + H_E + H_{\text{Coup}} \]

Projection

Lindblad master Eq. + quantum Diffusion
\[ \rho_S = |\phi\rangle\langle\phi| \]

Stoch. master Eq. + quantum Diff.
\[ D = \rho_S \otimes \rho_E \]
\[ \rho_S = |\phi_1\rangle\langle\phi_2| \]

Quantum Monte-Carlo (Exact)

Lindblad master Eq. + quantum Diffusion
\[ \rho_S = |\phi\rangle\langle\phi| \]

Stoch. master Eq. + quantum Diff.
\[ D = \prod \rho_i \]
\[ D = |\phi_1\rangle\langle\phi_2| \]


(G. Hupin talk)
More insight in mean-field dynamics:

**Exact state**
\[ |\Psi(t)\rangle \]

**Trial states**
\[ \{ |Q(t)\rangle, |Q + \delta Q\rangle = e^{\sum \delta q_\alpha A_\alpha} |Q\rangle \} \]

The approximate evolution is obtained by minimizing the action:
\[ S = \int_{t_0}^{t_1} ds \langle Q | i\hbar \partial_t - H | Q \rangle \]

**Included part: average evolution**
\[ \frac{d\langle A_\alpha \rangle}{dt} = \langle [A_\alpha, H] \rangle \]
\[ H = \mathcal{P}_1 H + (1 - \mathcal{P}_1) H \]

**Missing part: correlations**
\[ \langle dQ \rangle = \sum_{\alpha} dq_\alpha A_\alpha |dQ\rangle = \frac{dt}{i\hbar} \mathcal{P}_1(t) H |Q\rangle \]
\[ i\hbar \frac{d\langle A_\alpha A_\beta \rangle}{dt} \neq \langle [A_\alpha A_\beta, H] \rangle \]

**Hamiltonian splitting**
\[ H = \mathcal{P}_1 H + (1 - \mathcal{P}_1) H \]

The idea is now to treat the missing information as the **Environment** for the Relevant part (System).
Existence theorem: Optimal stochastic path from observable evolution


Theorem:
One can always find a stochastic process for trial states such that
\[ \langle A_a \rangle, \langle A_\alpha A_\beta \rangle, \cdots \langle A_{\alpha_1} A_{\alpha_2} \cdots A_{\alpha_k} \rangle \]
evolves exactly over a short time scale.

Valid for
\[ D = |Q_a \rangle \langle Q_b | \]
or
\[ D = \frac{|Q_a \rangle \langle Q_b |}{\langle Q_b | Q_a \rangle} \]

Mean-field level

\[
\begin{aligned}
\delta q^{[a]}_\alpha &= \delta q^a_\alpha \\
\delta q^{[b]}_\alpha^* &= \delta q^b_\alpha^*
\end{aligned}
\]

\[ i\hbar \frac{d}{dt} \langle A_\alpha \rangle = \langle [A_\alpha, H] \rangle \]

In practice

Mean-field + noise

\[
\begin{aligned}
\delta q^{[a]}_\alpha &= \delta q^a_\alpha + \delta q^{[2]}_\alpha \\
\delta q^{[b]}_\alpha^* &= \delta q^b_\alpha^* + \delta q^{[2]}_\alpha
\end{aligned}
\]

\[ i\hbar \frac{d}{dt} \langle A_\alpha \rangle = \langle [A_\alpha, H] \rangle \]

Exact evolution

Mean-field

\[ \langle A_1, A_2 \rangle, \langle A_2 \rangle \]
Mean-field evolution:

Reduction of the information:

I want to simulate the expansion with Gaussian wavefunction having fixed widths. \( \langle x^2 \rangle = cte, \quad \langle p^2 \rangle = cte \)

Relevant/Missing information:

**Relevant degrees of freedom**

- \( \langle x \rangle \), \( \langle p \rangle \)
- \( \langle a^+ \rangle \), \( \langle a \rangle \)

**Missing information**

- \( \langle x^2 \rangle \), \( \langle p^2 \rangle \), \( \langle xp \rangle \)
- \( \langle a^{+2} \rangle \), \( \langle a^2 \rangle \), \( \langle a^+ a \rangle \)

**Trial states**

- Coherent states

\[ |Q + \delta Q \rangle = e^{\sum \delta q_i A_i} |Q \rangle \]

\[ |\alpha + d\alpha \rangle = e^{d^2 a^+} |\alpha \rangle \]
Stochastic c-number evolution from Ehrenfest theorem

\[ D = \frac{\langle \beta | \rho | \beta \rangle}{\langle \beta | \beta \rangle} \quad \text{with} \quad \langle \beta + d\beta | \rho | \beta + d\beta \rangle = e^{d\beta^\ast d\eta} \langle \beta + d\beta | \rho | \beta + d\beta \rangle \]

\[
\begin{align*}
\frac{d\alpha}{d\alpha} &= d\bar{\alpha} + d\xi^2,
\frac{d\beta^*}{d\beta^*} &= \frac{d\bar{\beta}^* + d\eta^2}{d\beta^*},
\end{align*}
\]

mean values

\[
\begin{align*}
\frac{d\langle a \rangle}{d\langle a \rangle} &= \frac{d\bar{\alpha}}{d\bar{\alpha}},
\frac{d\langle a^2 \rangle}{d\langle a^2 \rangle} &= \frac{2\alpha d\alpha + d\xi^2 d\xi^2}{2\beta^* d\beta^* + d\eta^2 d\eta^2},
\end{align*}
\]

fluctuations

Nature of the stochastic mechanics

\[
\begin{align*}
X &= \frac{1}{\sqrt{2\eta}}(\alpha + \beta^*),
P &= i\hbar \sqrt{\frac{\eta}{2}}(\beta^* - \alpha)
\end{align*}
\]

\[
\begin{align*}
\frac{dX}{d\chi_1} &= \frac{P}{m} dt + d\chi_1,
\frac{dP}{d\chi_2} &= d\chi_2,
\end{align*}
\]

with \( \frac{d\chi_1}{d\chi_2} = \frac{\hbar^2 \eta}{2m} dt \)

the quantum wave spreading can be simulated by a classical brownian motion in the complex plane

Starting point:

\[ H = \sum_{i,j} \langle i|T|j \rangle a_i^+ a_j + \frac{1}{2} \sum_{ijkl} \langle ij|v_{12}|lk \rangle a_i^+ a_j^+ a_k a_l \]

\[ D_{ab} = |\Phi_a\rangle \langle \Phi_b| \quad \text{with} \quad \langle \Phi_b | \Phi_a \rangle = 1 \]

\[ \rho_1 = \sum_i |\alpha_i\rangle \langle \beta_i| \]

Ehrenfest theorem \(\Rightarrow\) BBGKY hierarchy

\[ i\hbar \frac{d}{dt} \rho_1 = [H_{MF}, \rho_1], \quad v_{12} = \sum_\lambda O_\lambda(1) O_\lambda(2) \]

\[ i\hbar \frac{d}{dt} \rho_{12} = [H_{MF}(1) + H_{MF}(2), \rho_{12}] \]

\[ + (1 - \rho_1)(1 - \rho_2) v_{12} \rho_1 \rho_2 - \rho_1 \rho_2 v_{12} (1 - \rho_1)(1 - \rho_2) \]

Observables \( \langle j | \rho_1 | i \rangle = \langle a_i^+ a_j \rangle \)

Fluctuations \( \langle ij | \rho_{12} | kl \rangle = \langle a_k^+ a_i^+ a_j a_l \rangle \)

Stochastic one-body evolution

\[ d\rho_1 = [H_{MF}, \rho_1] + \sum_\lambda d\xi_\lambda^{[2]}(1 - \rho_1) O_\lambda \rho_1 + \sum_\lambda d\eta_\lambda^{[2]} (1 - \rho_1) O_\lambda \rho_1 \]

with \( \frac{d\xi_\lambda^{[2]} d\xi_\lambda^{[2]}}{d\eta_\lambda^{[2]}} = -d\eta_\lambda^{[2]} d\eta_\lambda^{[2]} = \delta_{\lambda \lambda} \frac{dt}{i\hbar} \)

- The method is general.
  - The SSE are deduced easily
  - Extension to Stochastic TDHFB

- The mean-field appears naturally
  - and the interpretation is easier

- The numerical effort can be reduced by reducing the number of observables

---

SSE for Many-Body Fermions and bosons


Starting point:

\[ H = \sum_{i,j} \langle i|T|j \rangle a_i^+ a_j + \frac{1}{2} \sum_{ijkl} \langle ij|v_{12}|lk \rangle a_i^+ a_j^+ a_k a_l \]

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Summary, stochastic methods for Many-Body Fermionic and bosonic systems

Approximate evolution

<table>
<thead>
<tr>
<th>Mean-field</th>
<th>Simplified QJ</th>
<th>Generalized QJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D =</td>
<td>\Phi\rangle\langle\Phi</td>
<td>$</td>
</tr>
<tr>
<td>Fluctuation-</td>
<td>Fluctuation✓</td>
<td>Fluctuation✓</td>
</tr>
<tr>
<td>Dissipation</td>
<td>Dissipation</td>
<td>Dissipation✓</td>
</tr>
</tbody>
</table>

Exact QJ

$D = |\Phi_1\rangle\langle\Phi_2|$  
Everything✓

Variational QJ

$D = |Q_1\rangle\langle Q_2|$  
Partially everything✓

Numerical issues

Flexible

Fixed

Fixed

Flexible

Numerical instabilities