Decoherence in Low-Energy Nuclear Collisions?

Alexis Diaz-Torres

Department of Physics
• **Introduction**
  - Nuclear structure & collision dynamics
  - Coherent coupled channels model
  - Checking theory against measurements

• **Quantum decoherence in nuclear collisions**
  - Picture & main ideas
  - Decoherence in the complex-potential model?
  - Coupled-channels density-matrix approach

• **Summary**
Composite Atomic Nucleus

How do these excitations affect the nuclear collision dynamics?
Low-Energy Collision Dynamics: Coherent Quantum Description

Total potential

Coulomb Barrier

Internuclear distance

Energy

Relative motion

Intrinsic low-lying collective states

$|0\rangle$

$|1\rangle$

Wave-packet
Failure of the Coherent Quantum Description

Coupling Assisted Quantum Tunnelling: Nuclear Fusion

Measure of Fusion Probability

$^{16}\text{O} + ^{208}\text{Pb}$

Above the Barrier

Below the Barrier

Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604

Relative motion

Intrinsic quantum states
Quantum Decoherence in Nuclear Collisions

Quantum states: $|0\rangle$, $|1\rangle$, $|2\rangle$.

Master equation:
$$\frac{\partial \hat{\rho}}{\partial t} = [\hat{\mathcal{L}}_H + \hat{\mathcal{L}}_D] \hat{\rho}, \quad \hat{\rho}(0) = \hat{\rho}_0$$

Schrödinger description:
$$\hat{\mathcal{L}}_H \hat{\rho} = -i[\hat{\mathcal{H}}, \hat{\rho}]/\hbar$$

Decoherence & Absorption:
$$\hat{\mathcal{L}}_D \hat{\rho} = \sum_k \left( \hat{\mathcal{C}}_k \hat{\rho} \hat{\mathcal{C}}_k^\dagger - \frac{1}{2} \left[ \hat{\mathcal{C}}_k^\dagger \hat{\mathcal{C}}_k, \hat{\rho} \right]_+ \right)$$
$$\hat{\mathcal{C}}_{Ij} = \sqrt{\Gamma_{Ij}} |I\rangle \langle j|$$

In Practice (e.g., Pesce & Saalfrank, 1998):

$$\hat{\rho}(t) = \sum_{ij, rs} |i\rangle \rho_{ij}^{rs}(t) \langle j| s\rangle, \quad \rho_{ij}^{rs}(0) = \rho_{00}^{rs}(0) = g_0(r)g_0^*(s)$$

Intrinsic (energy) basis:
$$|i\rangle, i = 1, \ldots, N$$

Coordinate (grid) basis:
$$|r\rangle, r = 1, \ldots, M$$
Absence of Decoherence in the Optical Potential Model

AD-T, accepted as a Rapid Communication in PRC
Optical Potential Model

\[ \hat{\rho}_0 = |\chi_0\rangle \langle \chi_0| \]
\[ \dot{\hat{\rho}} = -\frac{i}{\hbar} (\hat{H}_{eff} \hat{\rho} - \hat{\rho} \hat{H}_{eff}^\dagger) \]

\[ \hat{H}_{eff} = \hat{H}_s - iW(x) \]
\[ \hat{H}_s = \hat{T} + V(x) \]
Optical Potential Model

\[
\rho_0(x, x') = \chi_0(x) \chi_0^*(x')
\]

\[
\dot{\rho}_{xx'} = -\frac{i}{\hbar} \left[ \hat{H}_s, \hat{\rho} \right]_{xx'} + (\mathcal{L}_D \hat{\rho})_{xx'},
\]

\[
\hat{H}_s = \hat{T} + V(x)
\]

\[
(\mathcal{L}_D \hat{\rho})_{xx'} = -\frac{1}{\hbar} \left( W(x) + W(x') \right) \rho_{xx'}
\]

Lindblad Dissipative Dynamics

\[
\begin{align*}
\dot{\rho}_{xx'}^{11} &= -\frac{i}{\hbar} \left[ \hat{H}_S, \hat{\rho} \right]_{xx'}^{11} + (\mathcal{L}_D \hat{\rho})_{xx'}^{11}, \\
\dot{\rho}_{xx'}^{22} &= (\mathcal{L}_D \hat{\rho})_{xx'}^{22}, \\
(\mathcal{L}_D \hat{\rho})_{xx'}^{kl} &= \delta_{kl} \sum_{j=1}^{2} \sqrt{\gamma_{xx}^{kj}} \rho_{xx'}^{jj} \sqrt{\gamma_{x'x'}^{kj}} \\
&- \frac{1}{2} \sum_{j=1}^{2} (\gamma_{xx}^{jk} + \gamma_{x'x'}^{jl}) \rho_{xx'}^{kl}, \\
\end{align*}
\]

The absorption rate to state \( |2\rangle \) is given by \( \gamma_{xx}^{21} = W(x)/\hbar \)
Measure of Coherence


For a pure state described by the state vector $|\chi\rangle$:

$\hat{\rho} = |\chi\rangle\langle\chi|$, and $\text{Tr}(\hat{\rho}) = \langle\chi|\chi\rangle$.

$\hat{\rho}^2 = |\chi\rangle\langle\chi|\langle\chi|\chi\rangle$, and $\text{Tr}(\hat{\rho}^2) = \langle\chi|\chi\rangle \langle\chi|\chi\rangle = [\text{Tr}(\hat{\rho})]^2$.

Hence, $\frac{\text{Tr}(\hat{\rho}^2)}{[\text{Tr}(\hat{\rho})]^2} = 1$, for nonzero values of $\text{Tr}(\hat{\rho})$.

For a mixed state, there is no single state vector describing the system:

$\frac{\text{Tr}(\hat{\rho}^2)}{[\text{Tr}(\hat{\rho})]^2} < 1$.

The transition from a pure state to a mixed state is caused by decoherence.
Absence of Decoherence in the Optical Potential Model

AD-T, accepted as a Rapid Communication in PRC
Absence of Decoherence in the Optical Potential Model

AD-T, accepted as a Rapid Communication in PRC

Decoherence significantly affects quantum tunnelling, and thus scattering as well.
Coupled-Channels Density-Matrix Approach

Environment

Nucleonic d.o.f.

Rotational d.o.f.

Reduced System

$^{154}\text{Sm}$

$^{16}\text{O}$
|\chi\rangle = \sum_{LJ M} \psi_{k_0} (r) |0 L; J M\rangle \Rightarrow \hat{\rho}_0 = |\chi\rangle \langle \chi |

\hat{\rho}_0 = \sum_{\alpha, \alpha', rs} |r \rangle \langle \alpha | \rho^{rs}_{\alpha \alpha'} (t = 0) \langle \alpha' | (s |,

where \alpha \equiv (IL; JM), |\alpha\rangle and |r\rangle are the coupled angular momentum basis and the discrete grid-basis describing the internuclear separations, respectively.

\rho^{rs}_{\alpha \alpha'} (t = 0) = N^2 \exp \left[ -\frac{(r - r_0)^2}{2\sigma^2} \right] e^{ik_0 r} \times \exp \left[ -\frac{(s - r_0)^2}{2\sigma^2} \right] e^{-ik_0 s} \delta_{I 0} \delta_{I' 0},

where \( N \) is determined from the normalization condition \( \sum_{r\alpha} \rho^{rr}_{\alpha \alpha} = 1 \).
Coupled-Channels Density-Matrix Approach

Equations of Motion

\[ i\hbar \dot{\rho}_{\alpha\alpha'}^{rs} = \sum_t (T^{rt} \rho_{\alpha\alpha'}^{ts} - \rho_{\alpha\alpha'}^{rt} T^{ts}) + [U_\alpha(r) - U_{\alpha'}(s)] \rho_{\alpha\alpha'}^{rs} + \sum_\beta [V_{\alpha\beta}(r) \rho_{\beta\alpha'}^{rs} - \rho_{\alpha\beta}^{rs} V_{\beta\alpha'}(s)] + (\varepsilon_\alpha - \varepsilon_{\alpha'}) \rho_{\alpha\alpha'}^{rs} + i\hbar \left\{ \delta_{\alpha\alpha'} \sum_\mu \sqrt{\Gamma_{\alpha\mu}^{rr}} \rho_{\mu\mu}^{rs} \sqrt{\Gamma_{\alpha\mu}^{ss}} - \frac{1}{2} \sum_\mu (\Gamma_{\mu,\alpha}^{rr} + \Gamma_{\mu,\alpha'}^{ss}) \rho_{\alpha\alpha'}^{rs} \right\} \]

\[ \dot{\rho}_{\bar{\alpha}\bar{\alpha}'}^{rs} = \delta_{\bar{\alpha}\bar{\alpha}'} \sum_\mu \sqrt{\Gamma_{\bar{\alpha}\mu}^{rr}} \rho_{\mu\mu}^{rs} \sqrt{\Gamma_{\bar{\alpha}\mu}^{ss}} - \frac{1}{2} \sum_\mu (\Gamma_{\mu,\bar{\alpha}}^{rr} + \Gamma_{\mu,\bar{\alpha}'}^{ss}) \rho_{\bar{\alpha}\bar{\alpha}'}^{rs} \]

Expectation value of an observable:
\[ \langle \hat{O}(t) \rangle = \frac{\text{Tr}[\hat{O} \hat{\rho}(t) \hat{O} \hat{\rho}(t)^\dagger]}{\text{Tr}[\hat{\rho}(t) \hat{\rho}(t)^\dagger]} \]
Coupled-Channels Density-Matrix Approach

Asymptotic Observables

The probability for producing the target in state \((I, M_I)\) with the relative coordinate in the direction \(\hat{r}'\):

\[
\frac{dW}{d\Omega}(I, M_I) = \sum_q C_{LmIM_I}^{JM} Y_{Lm}(\hat{r}') S_{\gamma\lambda}(t_f) \\
\times C_{L'm'M_I}^{J'M'} Y_{L'm'}^*(\hat{r}'),
\]

where \(q \equiv (L, m, J, M, L', m', J', M')\), \(\gamma \equiv (IL; JM)\), \(\lambda \equiv (IL'; J'M')\), and \(S_{\gamma\lambda}(t_f) = \sum_{r'} \rho_{r' r'}(t_f)\).

Integrating over all directions \(\hat{r}'\) of solid angles, and summing over all \(M_I\), the total probability for producing the target in state \(I\) (population) is obtained:

\[
W(I) = \sum_{M_I} \sum_{LmJM} (C_{LmIM_I}^{JM})^2 S_{\gamma\gamma}(t_f)
\]
Track decoherence and absorption through different environments. Environments are specific to particular degrees of freedom.
Application: Understanding fusion of astrophysically-important collisions at low energies

T. Spillane et al., PRL 98 (2007) 122501
E.F. Aguilera et al., PRC 73 (2006) 064601

Complex excitation modes in dinuclear system

AD-T, PRL 101 (2008) 122501
Neutron molecular shell structure of two interacting deformed $^{12}$C

\[ V = \sum_{s=1}^{2} e^{-iR_s \hat{k}} \hat{U}(\Omega_s) V_s \hat{U}^{-1}(\Omega_s) e^{iR_s \hat{k}} \]

\[ V_s \approx \sum_{\nu\mu} ^N \langle s\nu | V_{\nu\mu}^s | s\mu \rangle \]

AD-T, PRL 101 (2008) 122501
Summary

★ Coupled-channels density-matrix approach, which will quantify the importance of quantum decoherence in various areas of nuclear reaction theory.

★ Decoherence should always be explicitly included when modelling low-energy nuclear collision dynamics with a limited set of (relevant) degrees of freedom.
Application: Unified quantum description of reaction processes of neutron-rich, weakly-bound nuclei